SEMINAR: P-ADISCHE ZAHLEN

Prof. Dr. Salma Kuhlmann
Mitarbeiter: Dr. Lorna Gregory
2-stündig, Mo 11.45-13.15 Uhr, Raum M628

In the references below Serre will refer to Serre's book "A course in Arithmetic". The roman numerals refer to the chapter.

0. Topology, valuations and $\mathbb{Z}_p$ - Lorna 29th April
In this talk we will introduce basic notions from topology and construct $\mathbb{Z}_p$.

1. Hensel's Lemma - Niklas Müller 6th May
In this talk we will discuss lifting zeros in $\mathbb{Z}/p^n\mathbb{Z}$ of polynomials (with coefficients in $\mathbb{Z}_p$) to zeros in $\mathbb{Z}_p$.
Ref: Serre II §2

2. The multiplication group of $\mathbb{Q}_p$ and squares in $\mathbb{Q}_p$ - Arve Gengelbach 13th May
We will investigate the group of units of $\mathbb{Z}_p$, give a decomposition of $\mathbb{Q}_p$ as a multiplicative group and investigate squares in $\mathbb{Q}_p$.
Ref: Serre II §3

3. The Hilbert symbol I - Carmen Widera 3rd June
Let K be either $\mathbb{R}$ or $\mathbb{Q}_p$. The Hilbert symbol is the function from $K^\times \times K^\times$ to $\{1, -1\}$ defined as follows:

$$(a,b) := \begin{cases} 1, & \text{if the equations } z^2 = ax^2 + by^2 \text{ has a non-trivial solution in } K^3; \\ -1, & \text{Otherwise.} \end{cases}$$

We investigate the basic properties of this symbol.
Ref: Serre III §1
Knowledge of the Legendre Symbol/quadratic reciprocity will be required.

4. The Hilbert symbol II - Magdalena Körber 10th June
We prove the product formula for Hilbert symbols, show that $\mathbb{Q}$ is dense in $\prod_{p \in S} \mathbb{Q}_p$ for any finite set of primes S and use Dirichlet's theorem to show the existence of rational numbers with particular Hilbert symbols in $\mathbb{Q}_p$.
Ref: Serre III §2

Knowledge of the Legendre Symbol/quadratic reciprocity will be required.

5. Quadratic forms I - Charu Goel 17th June
We review basic notions for quadratic forms over fields.
Ref: Serre IV §1

6. Quadratic forms II - 24th June
We study quadratic forms over $\mathbb{Q}_p$.
Ref: Serre IV §2

7. Hasse-Minkowski - 1st July
We prove that a quadratic form with coefficients in $\mathbb{Q}$ has a non-trivial zero in $\mathbb{Q}$ if and only if it has a non-trivial zero in $\mathbb{R}$ and in $\mathbb{Q}_p$ for all primes p.
Ref: Serre IV §3