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Fachbereich
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Konstanz, den 5. Dezember 2012

Im

Oberseminar Partielle Differentialgleichungen

wird am

Donnerstag, dem 06. Dezember 2012,

folgender Vortrag gehalten:

PD Dr. rer. nat. habil. Ruben Jakob (Universität Tübingen):

"Sufficient conditions for Willmore-immersions in $\mathbb{R}^3$ to be minimal surfaces"

Zeit: 15:15 Uhr

Raum: F 426

Interessenten sind herzlich willkommen!

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Abstract: We provide two sharp sufficient conditions for immersed Willmore surfaces in $\mathbb{R}^3$, defined on finitely connected subdomains of $\mathbb{R}^2$, to be already minimal surfaces, i.e. to have vanishing mean curvatures on their entire domains. Our precise results read as follows:

Theorem 1. *Let $\Theta \subset \mathbb{R}^2$ denote some arbitrary domain, $X \in C^4(\Theta, \mathbb{R}^3)$ some immersed Willmore surface with unit normal vectorfield N satisfying $H \equiv 0$ on $\partial\Omega$ for some bounded, finitely connected C^3-domain Ω with $\overline{\Omega} \subset \Theta$. Furthermore, assume that there exist real constants c, d and some fixed vector $V \in \mathbb{R}^3 \backslash \{0\}$ such that the surfaces $\tilde{X} := cX + dV$ and X satisfy at least one of the following conditions:*

a) $H \geq 0$ (or $H \leq 0$) holds in $\Omega \cap O$, where $O \subset \mathbb{R}^2$ is some open neighbourhood of $\partial\Omega$, and

$$\langle \tilde{X}, N \rangle \geq 0 \quad \text{on} \quad \partial\Omega,$$
$$\langle \tilde{X}, N \rangle(x^*) > 0 \quad \text{in at least one point} \quad x^* \in \partial\Omega.$$

b)

$$\langle \tilde{X}, N \rangle \geq 0 \quad on\ \bar{\Omega},$$
$$\langle \tilde{X}, N \rangle > 0 \quad on\ \bar{\Omega} \setminus \mathcal{A},$$

for some compact subset $\mathcal{A} \subset \bar{\Omega}$ with $\mathcal{H}^1(\mathcal{A}) = 0$.

Then $H \equiv 0$ is satisfied in $\overline{\Omega}$, i.e. X is a minimal surface on $\overline{\Omega}$.

These results turn out to be particularly suitable for applications to Willmore graphs. We can therefore show that Willmore graphs on bounded, finitely connected C^3-domains $\overline{\Omega}$ with vanishing mean curvature on the boundary $\partial\Omega$ must already be minimal graphs, which in particular yields some Bernstein-type result for Willmore graphs on $\mathbb{R}^2$.

(invited by Prof. Dr. Oliver Schnürer)