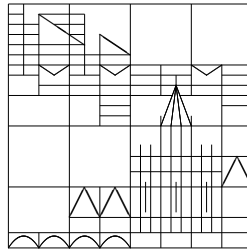


Universität Konstanz

Fachbereich
Mathematik und Statistik



Prof. Dr. Robert Denk
Prof. Dr. Michael Dreher
Prof. Dr. Reinhard Racke
Prof. Dr. Oliver Schnürer

Konstanz, den 5. Dezember 2012

Im

Oberseminar Partielle Differentialgleichungen

wird am

Donnerstag, dem 06. Dezember 2012,

folgender Vortrag gehalten:

PD Dr. rer. nat. habil. Ruben Jakob (Universität Tübingen):

“Sufficient conditions for Willmore-immersions in \mathbb{R}^3 to be minimal surfaces”

Zeit: 15:15 Uhr

Raum: F 426

Interessenten sind herzlich willkommen!

R. Denk, M. Dreher, R. Racke, O. Schnürer

Abstract: We provide two sharp sufficient conditions for immersed Willmore surfaces in \mathbb{R}^3 , defined on finitely connected subdomains of \mathbb{R}^2 , to be already minimal surfaces, i.e. to have vanishing mean curvatures on their entire domains. Our precise results read as follows:

Theorem 1. Let $\Theta \subset \mathbb{R}^2$ denote some arbitrary domain, $X \in C^4(\Theta, \mathbb{R}^3)$ some immersed Willmore surface with unit normal vectorfield N satisfying $H \equiv 0$ on $\partial\Omega$ for some bounded, finitely connected C^3 -domain Ω with $\bar{\Omega} \subset \Theta$. Furthermore, assume that there exist real constants c, d and some fixed vector $V \in \mathbb{R}^3 \setminus \{0\}$ such that the surfaces $\tilde{X} := cX + dV$ and X satisfy at least one of the following conditions:

a) $H \geq 0$ (or $H \leq 0$) holds in $\Omega \cap O$, where $O \subset \mathbb{R}^2$ is some open neighbourhood of $\partial\Omega$, and

$$\langle \tilde{X}, N \rangle \geq 0 \quad \text{on } \partial\Omega,$$

$$\langle \tilde{X}, N \rangle(x^*) > 0 \quad \text{in at least one point } x^* \in \partial\Omega.$$

b)

$$\langle \tilde{X}, N \rangle \geq 0 \quad \text{on } \bar{\Omega},$$

$$\langle \tilde{X}, N \rangle > 0 \quad \text{on } \bar{\Omega} \setminus \mathcal{A},$$

for some compact subset $\mathcal{A} \subset \bar{\Omega}$ with $\mathcal{H}^1(\mathcal{A}) = 0$.

Then $H \equiv 0$ is satisfied in $\bar{\Omega}$, i.e. X is a minimal surface on $\bar{\Omega}$.

These results turn out to be particularly suitable for applications to Willmore graphs. We can therefore show that Willmore graphs on bounded, finitely connected C^3 -domains Ω with vanishing mean curvature on the boundary $\partial\Omega$ must already be minimal graphs, which in particular yields some Bernstein-type result for Willmore graphs on \mathbb{R}^2 .