Criterion of total positivity of generalized Hurwitz matrices

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For a given set of real numbers $a_0, a_1, \dots, a_n$, and an integer M, $2 \leqslant M \leqslant n$, the following infinite matrix

$$H_M = \begin{pmatrix} a_{M-1} & a_{2M-1} & a_{3M-1} & \dots \\ a_{M-2} & a_{2M-2} & a_{3M-2} & \dots \\ \vdots & \vdots & \vdots & \\ a_0 & a_M & a_{2M} & \dots \\ 0 & a_{M-1} & a_{2M-1} & \dots \\ 0 & a_{M-2} & a_{2M-2} & \dots \\ \vdots & \vdots & \vdots & \\ 0 & a_0 & a_M & \dots \\ 0 & 0 & a_{M-1} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

is called generalized Hurwitz matrix, and for $M = 2$ the matrix H_2 is a standard *infinite* Hurwitz matrix. It is known [1, 5, 4, 2] that the total positivity of the matrix H_2 is equivalent to the positivity of the leading principal minors of H_2. In [3] (see also [6]), there were found finitely many sufficient conditions for the matrix H_M to be totally positive. In this talk, we show that positivity of finitely many certain minors of the generalized Hurwitz matrix H_M, $2 \leqslant M \leqslant n$, is necessary and sufficient for total positivity of the matrix H_M.

We also present some applications of totally positive generalized Hurwitz matrices to the root location of polynomials.

References

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