University of Konstanz
Department of Mathematics and Statistics
Winter Term 2016/2017

Sebastian Gruler
Markus Schweighofer

Fourier Analysis of Boolean Functions – Exercise Sheet 1

Exercise 1. Compute the Fourier expansions of the following functions:

(a) the logical "or" function on 3 bits OR_3

(b) the logical "and" function on 3 bits AND_3

(c) the majority function on 5 bits $\mathrm{Maj}_5\colon \{-1,1\}^5 \to \{-1,1\}$ that outputs the bit occurring more frequently in its input

(d) the sortedness function $\mathrm{Sort}_4\colon \{-1,1\}^4 \to \{-1,1\}$, defined by $\mathrm{Sort}_4(x) = -1$ if and only if $x_1 \le x_2 \le x_3 \le x_4$ or $x_1 \ge x_2 \ge x_3 \ge x_4$

Exercise 2. Prove that any $f\colon \{-1,1\}^n \to \{-1,1\}$ has at most one Fourier coefficient with absolute value bigger than $\frac{1}{2}$. Is this also true for any $f\colon \{-1,1\}^n \to \mathbb{R}$ with $\|f\|_2 = 1$?

Exercise 3. Given $f\colon \{-1,1\}^n \to \mathbb{R}$, define $F\colon \mathbb{R}^n \to \mathbb{R},\ x \mapsto \sum_{S\subseteq[n]} \widehat{f}(S)x^S$. Show that if $x \in [-1,1]^n$, then

$$F(x) = \mathbf{E}_{\mathbf{y}}[f(\mathbf{y})]$$

where $\mathbf{y} \in \{-1,1\}^n$ is a random variable such each $\mathbf{y}_i$ is chosen independently with $\mathbf{E}[\mathbf{y}_i] = x_i$.

Due Wednesday, November 2, 2016, 11:44 Uhr. Post it in the right box near room F411.