University of Konstanz
Department of Mathematics and Statistics
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Fourier Analysis of Boolean Functions – Exercise Sheet 2

Exercise 1. Let $f\colon \{-1,1\}^n \to \{-1,1\}$.

(a) Prove that f can not have exactly 2 nonzero Fourier coefficients. What about exactly 3 nonzero Fourier coefficients?

(b) Suppose $\|f_{\leq 1}\|_2^2 = 1$. How can f look like?

(c) Suppose $\|f_{\leq 2}\|_2^2 = 1$. Must f depend on at most 2 input coordinates? At most 3 input coordinates? What if we assume $\|f_{=2}\|_2^2 = 1$?

Exercise 2. Let $f\colon \{0,1\}^n \to \mathbb{R}$.

(a) Show that there exists a unique function $F\colon \mathscr{P}([n]) \to \mathbb{R}$ such that

$$f(x) = \sum_{S\subseteq[n]} F(S)x^S$$

for all $x \in \{0,1\}^n$.

(b) Show that F takes only integer values if f does so.

Exercise 3. Let $f\colon \{-1,1\}^n \to \{-1,1\}$ be of degree $d \geq 1$.

(a) Show that each Fourier coefficient of f is an integer multiple of 2^{1-d}.

(b) Show that $\sum_{S\subseteq[n]} |\widehat{f}(S)| \leq 2^{d-1}$.

Hint: Use Exercise 2.

Due Wednesday, November 16, 2016, 11:44 Uhr. Post it in box 18 near room F411.