University of Konstanz
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Winter Term 2016/2017

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Fourier Analysis of Boolean Functions – Exercise Sheet 5

Exercise 1. Consider a country (for example the USA) with m different states, indexed by $[m] = \{1, \dots, m\}$. For every state $i \in [m], s_i \in \mathbb{N}$ gives the number of people in this state, who are eligible to vote for the President of the country and let $N := \sum_{i=1}^{m} s_i$ be the number of people in the whole country, who are eligible to vote. There are two candidates for the Presidental election, indexed by $\{\pm 1\}$ and the country has the following voting rule: Every person in the country has one vote. The president is not elected by the people directly, but he is elected by the states. Every state i has $a_i \in \mathbb{N}$ votes, with the constraint that all votes of a state has to be for the one candidate, who has the most votes in this state.

(a) Let $f\colon \{-1,1\}^N \to \{-1,1\}$ the function that represents the voting rule of the country. Give an explicit formula for f. Therefor you can use known functions from the lecture.

(b) Which conditions of 2.1.2 are fullfilled by f for every choose of $m, s_1, \dots, s_m, a_1, a_a m \in \mathbb{N}$? Prove your statements.

Given now a model of a fictitious country with 5 states and 101 persons who are eligible to vote. The country has the voting rule mentioned above where the number of people who are eligible to vote in every state and the number of votes of every state are given by the following table.

state	1	2	3	4	5
people	11	25	37	7	21
votes	1	2	3	1	2

(c) Let f represent the voting rule of this country. For $1 \leq j \leq 101$ compute the Influence $\mathbf{Inf}_j[f]$ of every person in this country. Are your results logical explainable or predictable? Compare the Influences with the Incluence every person would have, if the President would be elected directly by the people via the Majority-vote.

Exercise 2. Let $w, s \in \mathbb{N}$ and write f for $\mathrm{Tribes}_{w,s}\colon \{-1,1\}^{ws} \to \{-1,1\}$. Compute $\mathbf{E}[f]$ and $\mathbf{I}[f]$.

Exercise 3. Let $f\colon \{-1,1\}^n \to \{-1,1\}$. It easily follows from Proposition 2.2.8 in the script that $\mathbf{I}[f] \leq \sqrt{n}$, if f is transitive-symmetric and monotone. Show that this also holds, if f is only monotone.

Exercise 4. Let $f\colon \{-1,1\}^n \to \{-1,1\}$ and $x \in \{-1,1\}^n$ and L the Laplacian operator defined as in Definiton 2.2.16 in the script.

(a) Show that $Lf(x) = f(x)\,\mathrm{sens}_f(x)$.

(b) Show that $\mathbf{I}[f] = \mathbf{E}_{\mathbf{S}\sim\widehat{f^2}}[\#S]$.

Due Wednesday, Januar 11, 2017, 11:44 Uhr. Post it in box 18 near room F411.