University of Konstanz
Department of Mathematics and Statistics
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Fourier Analysis of Boolean Functions – Exercise Sheet 6

Exercise 1. Let $f\colon \{-1,1\}^n \to \{-1,1\}$ be unbiased and let $\mathbf{MaxInf}[f] := \max_{i\in[n]} \mathbf{Inf}_i[f]$. Show $\mathbf{MaxInf}[f] \geq 1/n$.

Exercise 2. Let $f\colon \{-1,1\}^n \to \{-1,1\}$.

(a) Give an accurate proof of Remark 2.2.13 and deduce also very accurate $\mathbf{E}_{\mathbf{x}}[\mathrm{sens}_f(\mathbf{x})] = \mathbf{E}_{S\sim\hat{f}^2}[\#S]$.

(b) Show that also $\mathbf{E}_{\mathbf{x}}[\mathrm{sens}_f(\mathbf{x})^2] = \mathbf{E}_{S\sim\hat{f}^2}[(\#S)^2]$ holds.

(c) Is $\mathbf{E}_{\mathbf{x}}[\mathrm{sens}_f(\mathbf{x})^3] = \mathbf{E}_{S\sim\hat{f}^2}[(\#S)^3]$ also true?

Exercise 3. Let $f\colon \{-1,1\}^n \to \{-1,1\}$ a boolean function with the property $f(x) = f(-x)$ for all $x \in \{-1,1\}^n$.

(a) Show $\mathbf{Var}[f] \leq \frac{1}{2}\mathbf{I}[f]$.

(b) Show $\mathbf{Stab}_\rho[f] = \mathbf{Stab}_{-\rho}[f]$ for all $\rho \in [-1,1]$.

Exercise 4. Compute the precise probability of a Condorcet winner in a 3-candidate Condorcet election with 5 voters using $f = \mathrm{Maj}_5$ as voting rule and under the assumption that each of the 5 voters chooses uniformly and independently one of the 6 candidate rankings.

Due Friday, Januar 27, 2017, 11:44 Uhr. Post it in box 18 near room F411.