

Intermediate Cones between the SOS and PSD Cones

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Abstract

The cone of all real forms of degree $2d$ representable as finite sums of squares (SOS) of half degree d real forms is included in the cone of all positive semidefinite real forms (PSD) of even degree $2d$ for a fixed number of $n + 1$ variables. Hilbert [Hil88] states that these cones in fact coincide if and only if $n + 1 = 2$, $d = 1$ or $(n + 1, 2d) = (3, 4)$.

In this talk, we construct an explicit filtration of intermediate cones between the SOS and PSD cone by extending local positive semidefinite real quadratic forms along projective varieties generated by s ($s \geq 0$) real quadratic forms over the Veronese variety. Indeed, the Veronese variety is a projective variety finitely induced by real quadratic forms. We analyze this filtration for proper inclusions by using a result of Blekherman et al. [BSV16] on projective varieties of minimal degree, Hilbert's 1888 Theorem and techniques based on Robinson [Rob69] and Choi-Lam [CL77b, CL77a] exemplary in the quaternary quartics case.

References

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