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A constructive proof of the Helton-Vinnikov theorem

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This talk focuses on the Helton-Vinnikov theorem, which was shown in [3] to be equivalent to the celebrated Lax Conjecture. In 1958 Lax conjectured that any polynomial $p \in \mathbb{R}[x, y, z]$ of total degree $d \in \mathbb{N}$ being hyperbolic with respect to $(1, 0, 0) \in \mathbb{R}^3$ is representable as the determinant over a linear combination of two Hermitian matrices $Y, Z \in \mathbb{C}^{d \times d}$ and the identity matrix I_d i.e. $p(x, y, z) = \det(xI_d + yY + zZ)$. The Helton-Vinnikov theorem needs to be understood as a dehomogenized analogon of the Lax Conjecture and was first proven in [2], where the authors gave a quite technical proof. However, we will present the constructive proof of this theorem given in [1] and emphasize the idea of the proof using an example.

Bibliography

[1] A. Grinshpan, D. S. Kaliuzhnyi-Verbovetskyi, V. Vinnikov, H. J. Woerdeman, *Stable and Real-Stable Polynomials in two Variables*, Multidimensional Systems and Signal Processing 27 (2016), no.1, 1–26.

[2] J. W. Helton, V. Vinnikov, *Linear Matrix Inequalities Representation of Sets*, Communications on Pure and Applied Mathematics 60 (2007), no. 5, 654–674.

[3] A. S. Lewis, P. A. Parillo, M. V. Ramana, *The Lax Conjecture is True*, Proceedings of the American Mathematical Society, 133 (2005), no. 9, 2495–2499.