Universität Konstanz

Algebra Berichtseminar

organized by Dr. Maria Infusino and Prof. Dr. Salma Kuhlmann

Transferability results for topological properties of ordered groups and fields

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Let $\mathcal{L}_\mathrm{g} =< +, -, 0 >$ be the language of groups and let $\mathcal{L}_\mathrm{r} =< +, -, \cdot, 0, 1 >$ be the language of rings. For both we may add $\{<\}$ and obtain the language of ordered groups and rings, respectively. The study of definable valuations in certain fields motivates investigations concerning density and closedness of an ordered abelian group (or ordered field) in its divisible hull (or real closure). We will focus on the transferability of these properties, i.e. are they preserved under elementary equivalence in a certain language. Surprisingly, only in three out of four cases we loose transferability if $<$ is not at our disposal. We will give a brief overview of these three cases using results from [2], [3] and [1]. Thereafter we will gain deeper insights into the special case – closedness in the real closure (†) – showing that for an ordered field (†) is equivalent to being t-henselian as introduced in [4]. Using t-henselianity we will sketch the proof for $\mathcal{L}_\mathrm{r}$-transferability of (†). To conclude the talk we will construct a non-archimedean uniquely ordered field which is not dense in its real closure.

Bibliography

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[4] A. Prestel and M. Ziegler. *Model theoretic methods in the theory of topological fields.* J. reine angew. Math., Vol. 299/300, p. 318-341. 1978.