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A proof that every convex quarternary quartic form is a sum of squares

Moritz Schick

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In 1888 Hilbert managed to prove that the set $P_{n,2d}$ of positive semidefinite (PSD) forms and the set $\Sigma_{n,2d}$ of sum of square (SOS) forms in n variables of degree $2d$ coincide if and only if $n = 1$ or $d = 1$ or $(n, 2d) = (3, 4)$ holds. As we will see in this talk, every convex form of degree larger than 1 is already PSD. For this reason, Parillo asked in 2007, whether every convex form is SOS. Two years later, Blekherman managed to show that this is not the case, if the number n of variables is sufficiently large (see [1]). However, no explicit example of a convex but not SOS form is known until today. Due to Hilbert's characterization of PSD and SOS forms, the smallest $n, 2d$, where one could expect convex forms that are not SOS are $(n, 2d) = (3, 6)$ or $(n, 2d) = (4, 4)$. In this talk, we will focus on the latter case. Therefore, we follow the proof of El Khadir in [3] to see that for $(n, 2d) = (4, 4)$ indeed every convex quarternary quartic is SOS. We will see the key steps of his proof, which are the socalled Generalized Cauchy Schwarz Inequalities for convex forms as well as a characterization of the SOS cone $\Sigma_{4,4}$ inside the PSD cone $P_{4,4}$, which is based on Blekherman's work in [2].

Bibliography

[1] G. Blekherman: *Convex forms that are not sums of squares*, arXiv:0910.0656, (2009).

[2] G. Blekherman: *Nonnegative polynomials and sums of squares*, J. Amer. Math. Soc., 25 (2012), pp. 617-635.

[3] B. El Khadir: *On Sum of Squares Representation of Convex Forms and Generalized Cauchy-Schwarz Inequalities*, arXiv:1909.07546v1, (2019).