



# Master Fachseminar on Real Algebraic Geometry

organized by Dr. Maria Infusino and Prof. Salma Kuhlmann

## Uniform denominators in Hilbert's 17th Problem: Theorems by Polya and Reznick

*Robin Schabert*

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**Abstract.** Hilbert's 17th Problem was solved by Artin, who showed that every real positive semidefinite polynomial can be written as a sum of squares of rational functions. In this talk, we are going to present a related result due to Polya: For every homogeneous, positive definite and even polynomial  $f$  there is a  $k \in \mathbb{N}$ , s.t.  $(\sum X_i^2)^k f$  has only positive coefficients and is therefore a sum of squares of monomials, i.e. we can uniformly choose the denominators in Artin's solution. A similar result was proven by Reznick for polynomials not necessarily even together with a bound for the exponent  $k$ . We will prove the two above mentioned theorems and their non-homogenous versions. We will also discuss these results by giving examples in the positive definite case and a counterexample in the positive semidefinite one, and sketching some related open questions.