Interactions between Algebra, Geometry and Combinatorics

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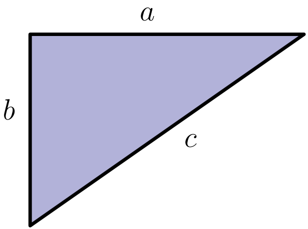
Pythagoras's theorem

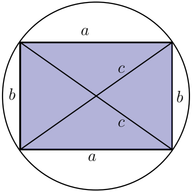


Theorem (Pythagoras, 570-495 BC)

The sides of a right triangle satisfy $a^2 + b^2 = c^2$.

Ptolemy's Theorem (Theorema Secundum)

A **cyclic quadrilateral** is a quadrilateral whose vertices lie on a common circle.



Theorem (Ptolemy, 70 - 168 BC)

The lengths of a sides and diagonals of a cyclic quadrilateral satisfy:
$|AC| \cdot |BD| = |AB| \cdot |CD| + |BC| \cdot |DA|$

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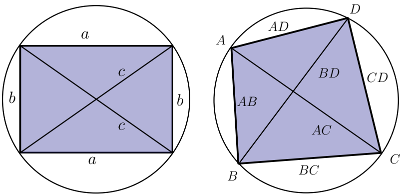

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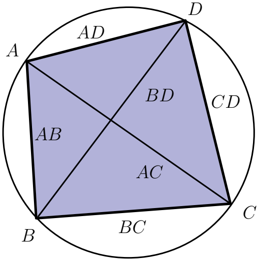

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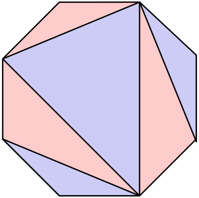
Triangulations

A **triangulation of an n-gon** is a subdivision of the polygon into triangles.



Result: $n-2$ triangles, using $n-3$ diagonals (**invariants** of the n-gon).

Theorem (Euler's Conjecture, 1751, Proofs: Catalan et al., 1838-39)

Number of ways to triangulate a convex n-gon:

$$C_n := \frac{1}{n-1}\binom{2n-4}{n-2}$$ **Catalan numbers**

n	3	4	5	6	7	8
C_n	1	2	5	14	42	132

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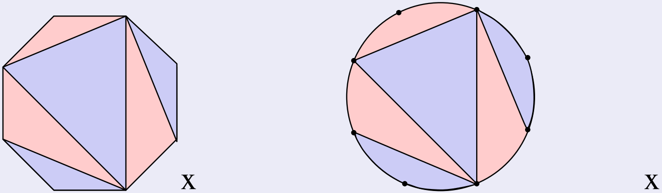
Figures in the plane: disk, annulus

Disk — Dynkin type A

n-gon: disk with n marked points on boundary.



Punctured disk — Dynkin type D

Marked points on boundary, one marked point in interior. (degenerate triangles)

Annulus — Dynkin type $\tilde{A}$

Marked points on both boundaries of the figure.

Finiteness (Fomin - Shapiro - D. Thurston 2005)

S Riemann surface

S is a disk and

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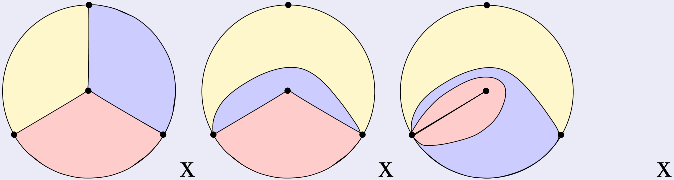

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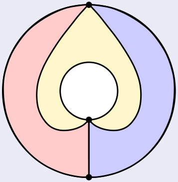
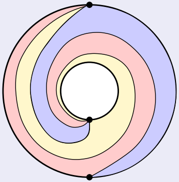


x



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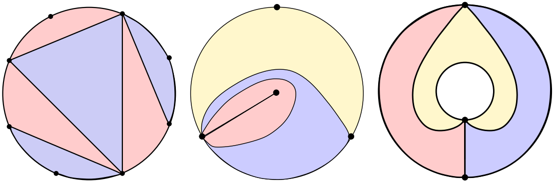
Finiteness (Fomin - Shapiro - D. Thurston 2005)

S Riemann surface with marked points M has finitely many triangulations $\iff$ S is a disk and M has at most one point in $S \setminus \partial S$

Triangles, diagonals



'Triangles'

Figures with three or less edges: degenerate triangles.

'Diagonals' (FST 2005)

The number of diagonals is constant. It is the **rank** of the surface:

$$p + 3q - 3(2 - b) + 6g,$$

where: p marked points, q punctures, b boundary components, g genus.

today: $q \in \{0, 1\}$, $b \in \{1, 2\}$, $g = 0$.

Cluster algebras

"In an attempt to create an algebraic framework for dual canonical bases and total positivity in semi simple groups, we initiate the study of a new class of commutative algebras."

Fomin-Zelevinsky 2001

Cluster Algebras

- recursively defined algebras $\subseteq \mathbb{Q}(x_1, \ldots, x_n)$
- grouped in overlapping sets of generators
- many relations between the generators

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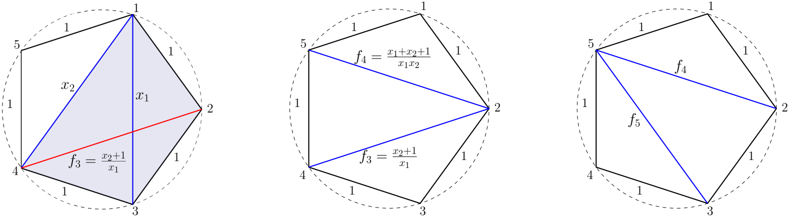
Pentagon-recurrence (Spence, Abel, Hill)

Sequence $(f_i)_i \subseteq \mathbb{Q}(x_1, x_2)$: $\quad f_{m+1} = \frac{f_m+1}{f_{m-1}}$ with $f_1 := x_1,\ f_2 := x_2$.

$f_3 = \frac{x_2+1}{x_1},\ f_4 = \frac{x_1+x_2+1}{x_1x_2},\ f_5 = \frac{x_1+1}{x_2}, \qquad f_6 = f_1,\ f_7 = f_2$, etc.



Cluster algebra $\mathcal{A} := \langle (f_i)_i \rangle = \langle f_1, f_2, \dots, f_5 \rangle \subseteq \mathbb{Q}(x_1, x_2)$.

Relations: $f_1f_3 = f_2 + 1,\ f_2f_4 = f_3 + 1$, etc.

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Cluster algebras

Start with $\{x_1, \ldots, x_n\}$ **cluster**, $B = (b_{ij})$ $n \times n$ a sign-skew symmetric matrix over $\mathbb{Z}$. The pair $(\underline{x}, B)$ is a seed. Relations through **mutation at k** (B mutates similarly):

$$x_k \cdot x_k' = \prod_{b_{ik}>0} x_i^{b_{ik}} + \prod_{b_{ik}<0} x_i^{-b_{ik}}$$

Mutation at k: $x_k \longmapsto x_k'$, and so $(\underline{x}, B) \rightsquigarrow (\{x_1, \ldots, x_k', \ldots, x_n\}, B')$.

Cluster variables all the x_i, the x_i', etc.

Cluster algebra $\mathcal{A} = \mathcal{A}(\underline{x}, B) \subset \mathbb{Q}(x_1, \ldots, x_n)$ generated by all cluster variables.

Cluster algebras

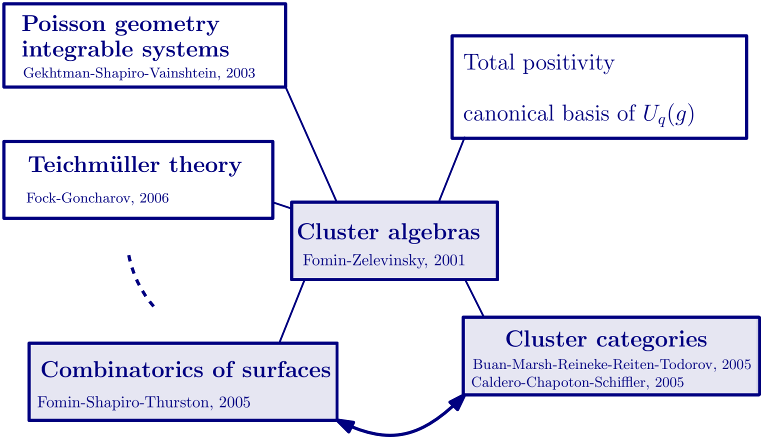
$\mathcal{A} = \mathcal{A}(\underline{x}, B) \subset \mathbb{Q}(x_1, \ldots, x_n)$.

Properties

- Laurent phenomenon: $\mathcal{A} \subset \mathbb{Z}[x_1^{\pm}, x_2^{\pm}, \ldots, x_n^{\pm}]$: Fomin-Zelevinsky.
- Finite type follows Dynkin type: Fomin-Zelevinsky.
- Positivity: coefficients in $\mathbb{Z}_{>0}$: Musiker-Schiffler-Williams 2011, Lee-Schiffler 2015, Gross-Hacking-Keel-Kontsevich 2018.

Examples
$\mathbb{C}[\mathrm{SL}_2]$, $\mathbb{C}[\mathrm{Gr}(2, n)]$ (Fomin-Zelevinsky), $\mathbb{C}[\mathrm{Gr}(k, n)]$ (Scott).

Overview



Poisson geometry
integrable systems
Gekhtman-Shapiro-Vainshtein, 2003
Total positivity
canonical basis of $U_q(g)$
Teichmüller theory
Fock-Goncharov, 2006
Cluster algebras
Fomin-Zelevinsky, 2001
Cluster categories
Buan-Marsh-Reineke-Reiten-Todorov, 2005
Caldero-Chapoton-Schiffler, 2005
Combinatorics of surfaces
Fomin-Shapiro-Thurston, 2005

In focus

Cluster algebras

Gratz, 2016
Parsons, 2015
Parsons, 2013
Grabowski-Gratz, 2014

B-Marsh w Thomas, 2009

B-Marsh 2012

Surface combinatorics

B-Dupont, 2014
Tschabold, arXiv 2015
B-Parsons-Tschabold, 2016
Vogel, 2016
Aichholzer-Andritsch-B-Vogtenhuber, 2017
Gunawan-Musiker-Vogel, 2018
B-Fellner-Parsons-Tschabold, 2018
B-Martin, 2018
B-Gekhtman*
B-Martin, arXiv 2017
B-Coelho Simoes, arXiv 2018
B-Marsh*
B-Schroll*

B-Marsh 2007, 2008, 2012
B-Buan-Marsh, 2014
B-King-Marsh, 2016
Lamberti, 2014
B-Torkildsen, arXiv 2015
B-Gratz, 2018
McMahon, arXiv 2016, 2017
Coelho Simoes-Parsons 2017
B-Faber-Gratz-Serhiyenko-Todorov, 2017
Andritsch, 2018
B-Faber-Gratz-Serhiyenko-Todorov*
B-Laking *
B-Beil *

Cluster categories

Lamberti, 2011, 2012
Gratz, 2015
B-Bogdanic, 2016
B-Bogdanic-García Elsener*
B-Coelho Simoes-Pauksztello*
B-Bogdanic-Pressland*
B-Bogdanic-García Elsener-Martsinkovsky *
B-Schiffler *
B-Nasr Isfahani *

$*$ ongoing projects

Background: algebra $\longleftrightarrow$ geometry

pentagon recurrence

cluster variables: $f_{m+1} = \frac{f_m+1}{f_{m-1}}$

$$x_2 \qquad \frac{x_1+x_2+1}{x_1x_2} \qquad x_1$$

$$x_1 \qquad \frac{x_2+1}{x_1} \qquad \frac{x_1+1}{x_2} \qquad x_2$$

Ptolemy relation

diagonals: $ac = ab + cd$

$$(1,4) \qquad (2,5) \qquad (1,3)$$

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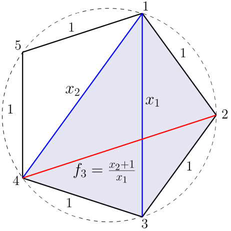

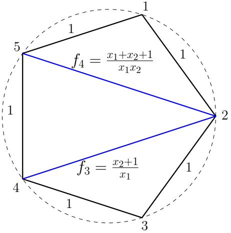

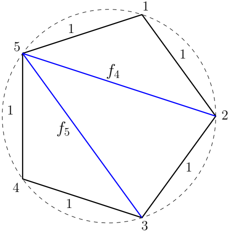

Analogies

- Polygon $\longleftrightarrow$ cluster algebra: Fomin-Zelevinsky, FST

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- Polygon $\longleftrightarrow$ cluster algebra: Fomin-Zelevinsky, FST
- Polygon $\longleftrightarrow$ cluster category: Caldero-Chapoton-Schiffler, B-Marsh
- motivates: Surfaces $\leftrightsquigarrow$ categories: B-King-Marsh

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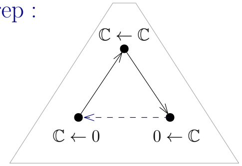
Cluster category

Definition (Cluster category of type Q, in example: $Q = 1 \leftarrow 2$)

$\mathcal{C}_Q := \mathrm{D}^b(Q\text{-rep})/(\tau^{-1} \circ [1])$

$a)$ Q - rep :



Category $\mathcal{C}_Q$:

Fin. many $\underbrace{\text{indecomposable objects}}_{\text{vertices}}$, arrows: irreducible morphisms.

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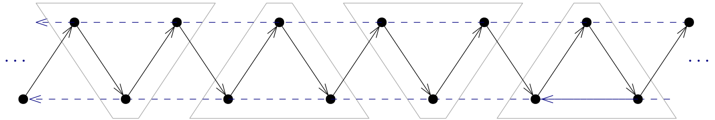
$b)$ $D^b(Q - \text{rep})$:



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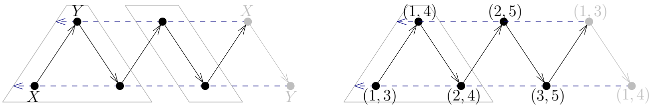

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Properties of cluster categories

Properties

- $\mathcal{C}_Q$ is Krull-Schmidt, triangulated, of Calabi-Yau dimension 2: Buan-Marsh-Reineke-Reiten-Todorov 2005, Keller, 2005.
- $\mathcal{C}_{A_{n-3}}$, $\mathcal{C}_{D_n}$ arises from triangulations of (punctured) n-gon: Caldero-Chapoton-Schiffler 2005, Schiffler 2008.
- m-cluster categories in types A, D from m-angulations of (punctured) polygons: B-Marsh 2007, 2008.

Strategy

Use combinatorial geometry to describe cluster categories:
Surface combinatorics yields cluster algebras, cluster categories.

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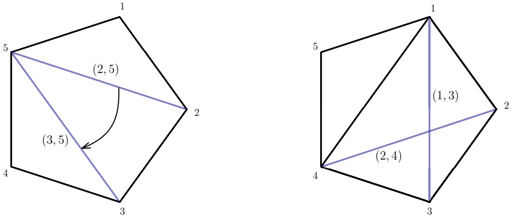
Categories via surfaces: 2 approaches

Approach 1:

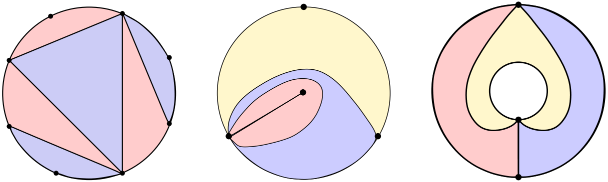
$\mathcal{C}_{A_2}$ described via diagonals in pentagon; $\mathcal{C}_{A_n}$ via diagonals in $(n+3)$-gon.

Objects correspond to diagonals. Morphisms via rotation. Extensions via intersections of diagonals, shift, τ, etc.



In $\mathcal{C}_{A_2}$: indecomposable objects X_d where d is a diagonal in pentagon. Irreducible morphism $X_{(2,5)} \to X_{(3,5)}$ and $\mathrm{Ext}^1_{\mathcal{C}}(X_{(1,3)}, X_{(2,4)}) \neq 0$.

Algebra via geometry: Approach 1



Development

Categories $\mathcal{C}_{D_n}$, $\mathcal{C}_{\tilde{A}_n}$, $\mathcal{C}_{E_6}$, module categories, root category, derived category etc.

B-Marsh [2007,08], Schiffler [2008], Warkentin [2009], Brüstle-Zhang [2011], Lamberti [2011,12], B-Marsh [2012], Coelho-Simoes [2012], Holm-Jørgensen [2012], Igusa-Todorov [2012,13], B-Dupont [2014], B-Torkildsen [2015], Gratz [2015], Torkildsen [2015], Canakci-Schroll [2017], B-Gratz [2018], Opper-Plamondon-Schroll [2018], B-Coelho Simoes [2018] etc.

Dimers on surfaces: Approach 2

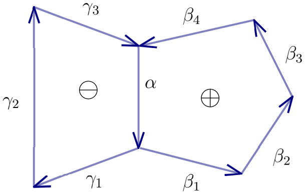
Triangulations of disks $\rightsquigarrow \mathcal{C}_{A_n}$ or $\mathcal{C}(\mathrm{Gr}(2, n+3))$.

New: can get $\mathcal{C}(\mathrm{Gr}(k, n))$ for any k through surface combinatorics.

Approach 2: B-King-Marsh 2016

The Grassmannian cluster category $\mathcal{C}(Gr(k, n))$ is described via (k, n)-dimers.

A **dimer** is an oriented graph (quiver) embedded in a surface, such that its complement is a union of disks. Comes with a natural potential.



$p_1(\alpha) = \beta_1\beta_2\beta_3\beta_4$

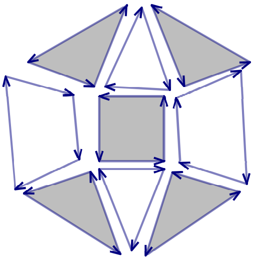
$p_2(\alpha) = \gamma_1\gamma_2\gamma_3$

$R_\alpha : \; p_1(\alpha) - p_2(\alpha)$

Potential: yields the relations R_α

A dimer with boundary (B-King-Marsh)

A **dimer with boundary** Q is a quiver embedded in a surface, glued from oriented disks. Arrows appear in 1 (boundary arrows) or 2 faces (interior).



A (k,n)-dimer

A dimer Q is a **(k,n)-dimer** if the surface is an n-gon and if the associated zig-zag paths induce the permutation $i \mapsto i+k$ from S_n.

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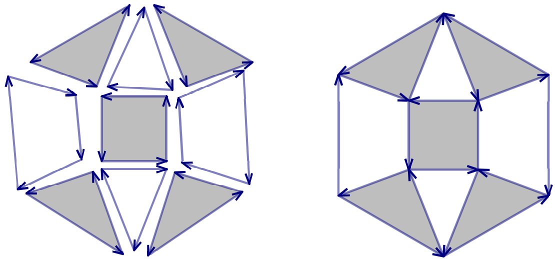

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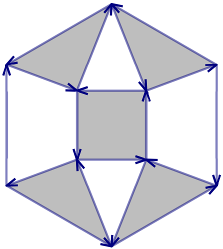

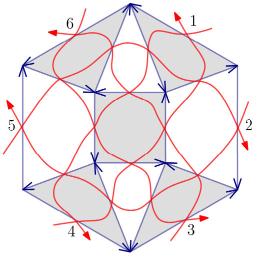

In red: zig-zag paths

$\sigma = (14)(25)(36)$
$(k, n) = (3, 6)$

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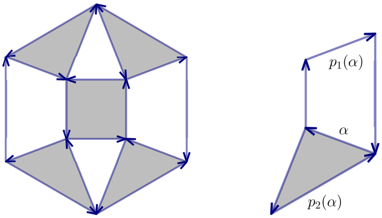
Dimers and associated algebras

Definition (Dimer algebra of a dimer Q)

Dimer algebra Λ_Q of Q: path algebra $\mathbb{C}Q$ with relations $p_1(\alpha) = p_2(\alpha)$.



Let $e_b \in \Lambda_Q$ be the sum of the trivial paths of the boundary vertices of Q.

Definition

Let Q be a dimer, with dimer algebra Λ_Q.
The **boundary algebra of Q** is the algebra $A_Q = e_b \Lambda_Q e_b$.

Dimers and associated algebras

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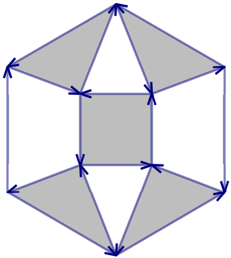
Example: $(3,6)$-dimer and its boundary algebra



Boundary algebra $A_Q = e_b \Lambda_Q e_b$ of $(3,6)$-dimer Q

Generators x_i, y_i, $i = 1, \ldots, 6$.
Relations: $xy = yx$, $x^3 = y^3$ (at each vertex) or $x^k = y^{n-k}$, respectively

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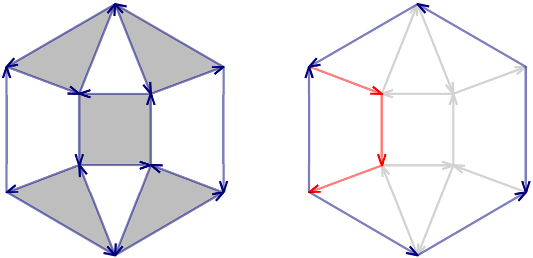

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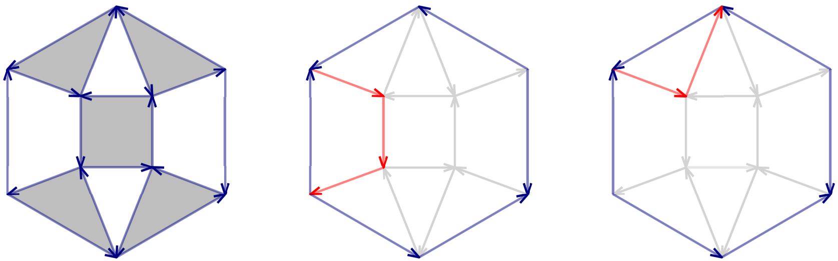

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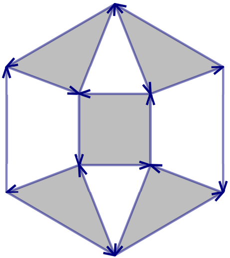

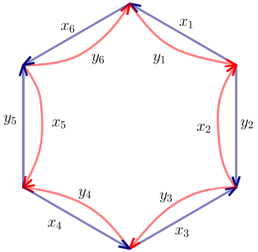

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(k,n)-dimers $\leftrightsquigarrow$ Grassmannians

Theorem (B-King-Marsh 2016)

Any (k,n)-dimer yields the cluster category $\mathcal{C}(Gr_{k,n})$.

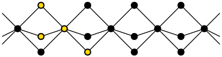
Key ingredients

- Let Q, Q' two (k,n)-dimers, with boundary algebras A_Q and $A_{Q'}$. Then $A_Q \cong A_{Q'}$ (B-King-Marsh 2016).
- $A_Q \cong B$, algebra used by Jensen-King-Su 2016.
- $CM(B)$ categorifies Scott's cluster structure of the Grassmannian cluster algebra $\mathbb{C}[\mathrm{Gr}(k,n)]$, Jensen-King-Su 2016.
- In $CM(A_Q)$, Plücker correspond to rank 1 modules.
- (k,n)-dimers yield cluster-tilting objects.

Results from (k, n)-dimers $\leftrightsquigarrow$ Grassmannians

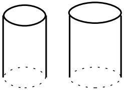
(k, n)-dimers yield $\mathcal{C}(\mathrm{Gr}(k, n))$

B-King-Marsh 2016



$\mathcal{C}(\mathrm{Gr}(3,6))$

tubular types from $\mathcal{C}(\mathrm{Gr}(k, n))$



B-Bogdanic-Garcia Elsener 2018

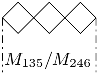
SL_k-friezes from $\mathcal{C}(\mathrm{Gr}(k, n))$

B-Faber-Gratz-Serh.-Todorov 2018

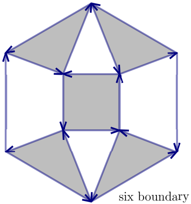
$$\begin{array}{ccccccccccc}
 & 1 & & 1 & & 1 & & 1 & & 1 & \\
1 & & 3 & & 1 & & 2 & & 2 & & 1 \\
 & 2 & & 2 & & 1 & & 3 & & 1 & \\
1 & & 1 & & 1 & & 1 & & 1 & & 1
\end{array}$$

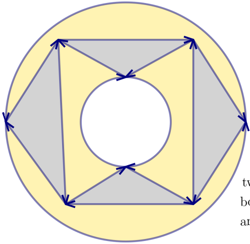
extensions in $\mathcal{C}(\mathrm{Gr}(k, n))$ via rims

B-Bogdanic 2017



Projects around dimers with boundary

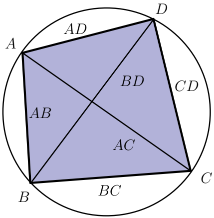




- Understand dimer on general surfaces with boundary.
- Develop theory of boundary algebras, of associated module categories.
- Laminations, starting with (k, n)-dimers.
- Friezes, SL_k-friezes.
- Dimer algebras and homotopy dimer algebras.
- Degenerate (k, n)-dimers. $k = 2$: tilings.

Approach 1

Ptolemy's theorem



In polygon

Cluster category $\mathcal{C}_{A_2}$

Polygon $\longleftrightarrow$ cluster category

$(1,4)$ and $(2,5)$ intersect, $\quad \mathrm{Ext}^1(X_{(1,4)}, X_{(2,5)}) \neq 0,$
$(2,4)$ and $(2,5)$ don't intersect. $\quad \mathrm{Ext}^1(X_{(2,4)}, X_{(2,5)}) = 0.$

Geometric interpretation of algebraic phenomena (e.g. [1], τ). Also for general surfaces and categories.

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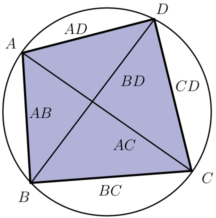

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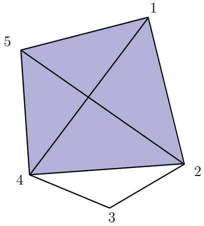

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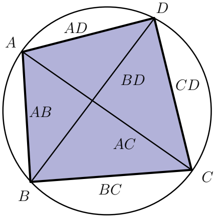

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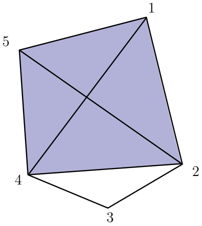

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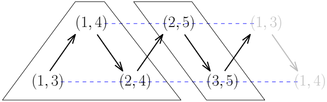

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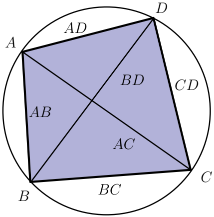

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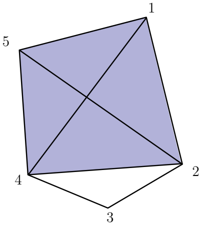

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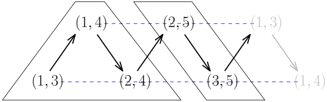

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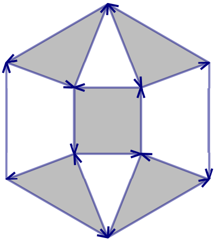
Approach 2

$(3, 6)$-dimer



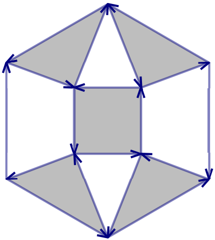
boundary algebra $\qquad \mathcal{C}(\mathrm{Gr}(3, 6))$

Dimer $\longleftrightarrow$ Category of modules

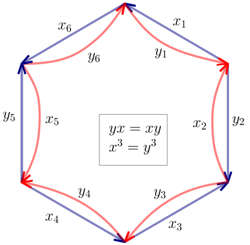
vertices	indecomposable objects
boundary vertices	projective-injective indec's

Approach 2

(3, 6)-dimer



boundary algebra



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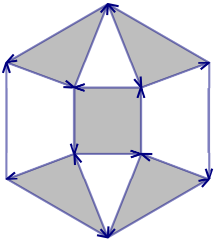
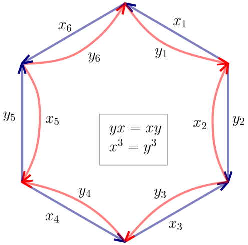


boundary algebra



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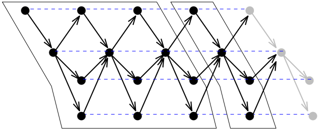

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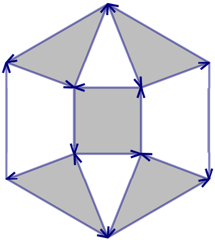

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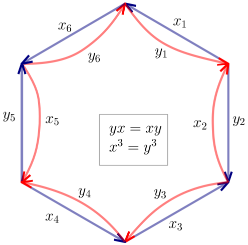

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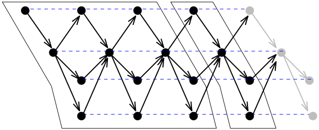

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CV

Karin Baur, PhD in Mathematics, born in Zurich.

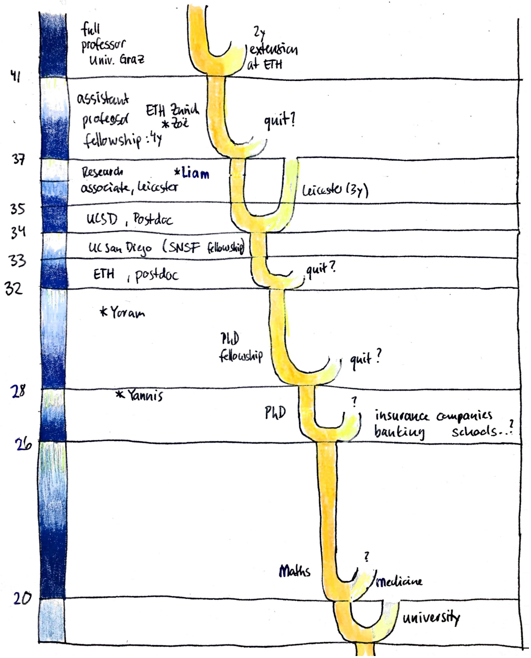
Education

1977 - 1989	Schools in Zurich, Matura type B (classical).
1990 - 1996	Studies (Mathematics, Philosophy, French Literature), University of Zurich.
1994	Erasmus (Paris VI).
1997 - 2001	PhD studies in Mathematics, University of Basel.

CV, continued

Academic positions

2002 - 2003	Post-doctoral assistant, ETH Zurich.
2003 - 2005	Post-doc (SNSF) University of California, San Diego, USA
2005 - 2007	Research associate (EPSRC), University of Leicester
2007 - 2011	Assistant professor (SNSF Professor), ETH Zurich
2011 -	Full professor, University of Graz, Austria
2018 -	Professor, University of Leeds, UK.


full professor Univ. Graz
2y extension at ETH
41
assistant professor fellowship: 4y
ETH Zurich
*Zoë
quit?
37
Research associate, Leicester
*Liam
Leicester (3y)
35
UCSD, Postdoc
34
UC San Diego (SNSF fellowship)
33
ETH, postdoc
quit?
32
*Yoram
PhD fellowship
quit?
28
*Yannis
PhD
?
insurance companies
banking
schools..?
26
Maths
?
medicine
20
university

Doubts and strategies

Questions along the way

- Are there jobs?
- Unknown area: new maths, new questions. Which direction?
- Endurance, patience.
- Geographically: where?
- What are the alternatives?

Resources

- Family
- Peers, friends
- Mentors
- Collaborators
- Deadlines
- Interest

Aspects of the job of a mathematician