Robust Solution Approaches for Optimization under Uncertainty: Applications to Air Traffic Management Problems

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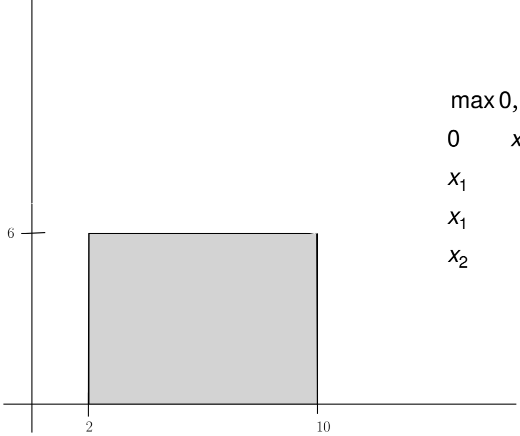
Konstanz, 15.11.2016

Relevance of Uncertainties in Optimization



$$
\begin{aligned}
\max\ & 0,7x_1 + x_2 \\
& 0\,x_1 + 1x_2 \leq 6 \\
& x_1 - 0\,x_2 \leq 10 \\
& x_1 \geq 2 \\
& x_2 \geq 0
\end{aligned}
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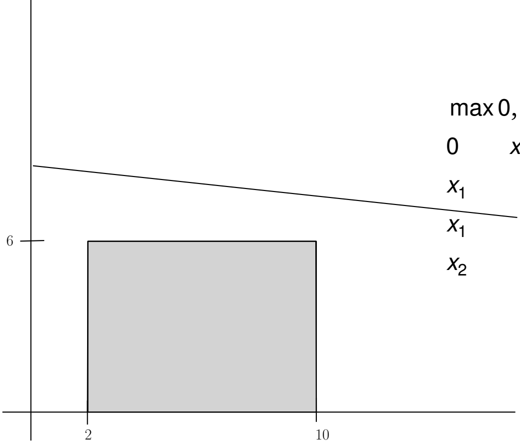

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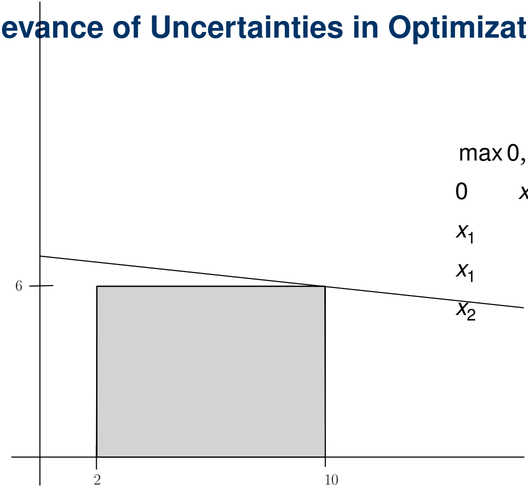

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\begin{array}{llll}
\max 0,7x_1 & +x_2 & & \\
0 \quad x_1 & +1x_2 & \leq 6 \\
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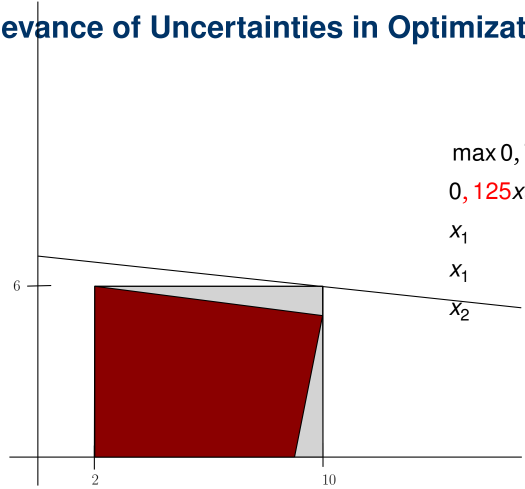

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\begin{aligned}
\max\ 0,7x_1 & + x_2 \\
0,125x_1 & + 1x_2 \leq 6,25 \\
x_1 & - 0,1\bar{6}x_2 \leq 10-1 \\
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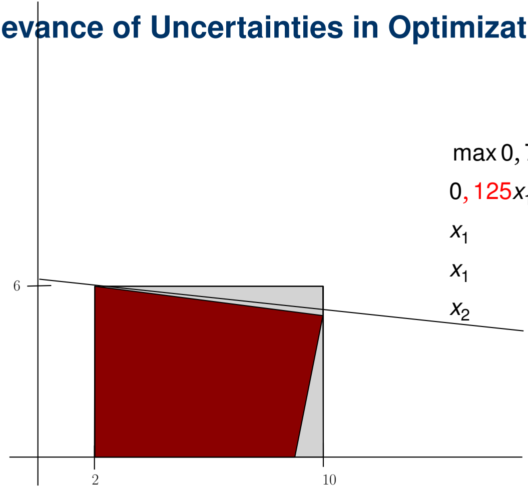

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Optimization Under Uncertainty

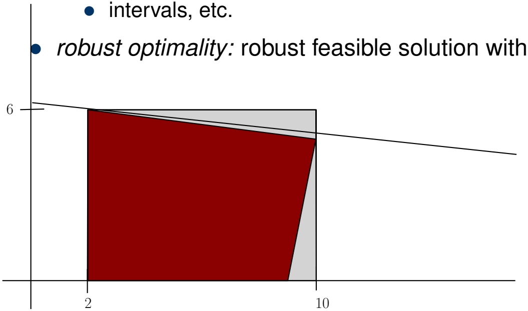
- just ignore, solve nominal problem

Optimization Under Uncertainty

- just ignore, solve nominal problem
- ex post: sensitivity analysis
- ex ante:
  - stochastic optimization
  - *robust optimization*

Protection Against the Worst Case

- *robust feasibility:* solution has to be feasible for all inputs against protection is sought
- beforehand, define *uncertainty set U*:
  - based on scenarios, or
  - intervals, etc.
- *robust optimality:* robust feasible solution with best guaranteed solution value



Robust versus Stochastic Optimization

robust optimization	*stochastic optimization*
worst-case	expected value
uncertainty sets	probability distributions
100 % protection against pre-defined uncertainty set U	protection with certain probability
when what? distributions unknown	 distributions known
“probably” is not enough	expectated value sufficient

Robust versus Stochastic Optimization

robust optimization	*stochastic optimization*
worst-case	expected value
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100 % protection against pre-defined uncertainty set U	protection with certain probability
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evaluation with respect to

- mathematical tractability
- conservatism of the solution

Air Traffic Management

Fürstenau (DLR), Heidt, Kapolke, Liers, Martin, Peter, Weiss (DLR)

- continous growth of traffic demand
- possibilities of enlarging airport capacities are limited

source: tagaytayhighlands.net

→ efficient utilization of existing capacities is crucial

Optimization of **runway utilization** is one of the main challenges in ATM.

Outline

- pre-tactical and tactical planning planning: time-window assignment and runway scheduling
- for both planning phases: affect of uncertainties, and
- protection against uncertainties using robust optimization

Pre-tactical Planning

$=$ *a considerable amount of time prior to scheduled arrival times*

$\rightarrow$ don't need to determine exact times/sequence yet

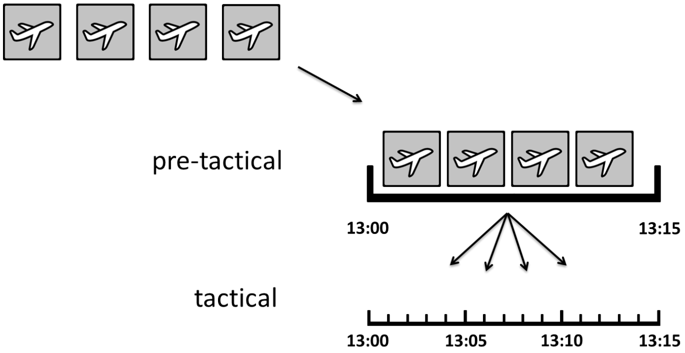
Idea:

assign several aircraft to one time window of a given size (e.g. 15 min)

$\longrightarrow$ omit unnecessary information

$\longrightarrow$ reduce complexity

Nominal Problem: Time-Window Assignment



Time-Window Assignment

- each aircraft has to receive **exactly one** time window
- each time window can be assigned to **several** aircraft

Questions:

1) Which time windows can be assigned to which aircraft?
2) How many aircraft fit in one time window?

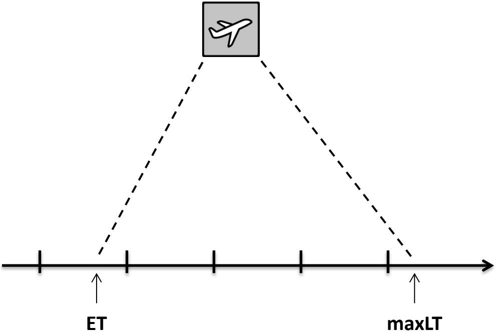
Which Time Windows can be Assigned to Which Aircraft?

Each aircraft has its individual...

ST	= scheduled time of arrival (flight plan)
ET	= earliest time of arrival (dependent on operational conditions)
LT	= latest time of arrival (without holdings) (dependent on ET)
maxLT	= maximal latest time of arrival (dependent on amount of fuel etc.)

...and thus can be assigned to time windows between **ET** and **maxLT**.

Which Time Windows can be Assigned to Which Aircraft?



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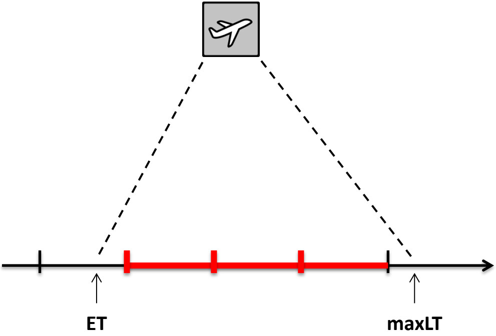

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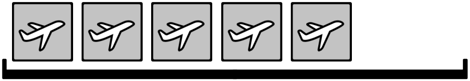
How Many Aircraft can be Assigned to One Time Window?

given a set of aircraft: do they fit in the same time window?



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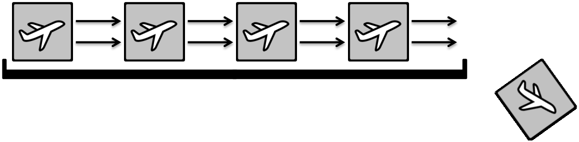
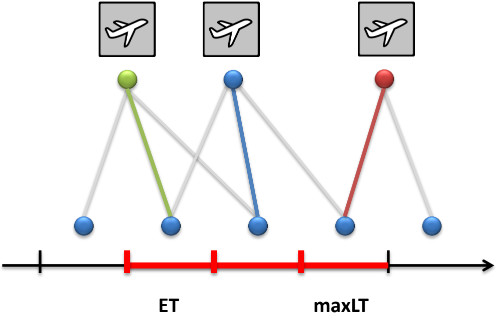


satisfy distance requirements

Time-Window Assignment Graph



- **assignment decisions:** in *b*-matching problem

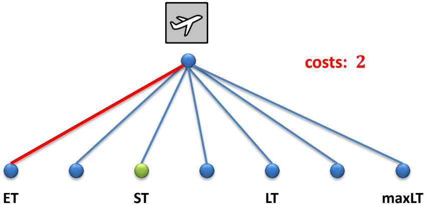
$\longrightarrow$ binary variables $x_{ij} = \begin{cases} 1, & \text{if aircraft } i \text{ is assigned to time window } j \\ 0, & \text{otherwise} \end{cases}$

Time-Window Assignment: Objective

maximize punctuality

i.e. **minimize** deviation from scheduled times (delay and earliness)

- earliness is penalized linearly
- delay is penalized quadratically, for reasons of fairness:
  one aircraft with large delay is worse than two aircraft with little delay
- extra penalization term for time windows between LT and maxLT



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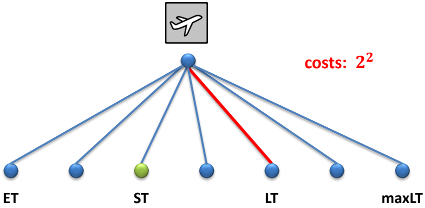

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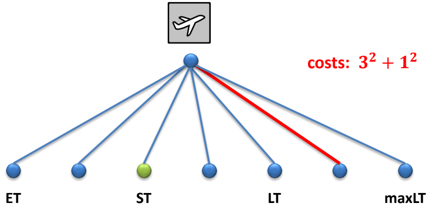


$$\min \sum_{(i,j)\in E} c_{ij} x_{ij}$$

s.t. Exactly one time window for each aircraft

Distance requirements in each time window

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in E$$

$$\min \sum_{(i,j)\in E} c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j\in W_i} x_{ij} = 1 \qquad \forall i \in A \qquad (1)$$

Distance requirements in each time window

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in E$$

- basically yields a *b*-matching problem (with side constraints)
- ...when incorporating different separation times according to weight classes...

$$\min \sum_{(i,j)\in E} c_{ij}x_{ij}$$

$$\text{s.t.} \quad \sum_{j\in W_i} x_{ij} = 1 \qquad \forall i \in A \qquad (1)$$

$$75\sum_{i\in L_j} x_{ij} + 75\sum_{i\in M_j} x_{ij} + 100\sum_{i\in H_j} x_{ij} + 100z_j^{HH} \leq s + 100 \qquad \forall j \in W\setminus\{m\} \qquad (2)$$

$$75\sum_{i\in L_j} x_{ij} + 75\sum_{i\in M_j} x_{ij} + 100\sum_{i\in H_j} x_{ij} + 125z_j^{HM} \leq s + 100 \qquad \forall j \in W\setminus\{m\} \qquad (3)$$

$$75\sum_{i\in L_j} x_{ij} + 75\sum_{i\in M_j} x_{ij} + 100\sum_{i\in H_j} x_{ij} + 150z_j^{HL} \leq s + 100 \qquad \forall j \in W\setminus\{m\} \qquad (4)$$

$$75\sum_{i\in L_j} x_{ij} + 75\sum_{i\in M_j} x_{ij} + 100\sum_{i\in H_j} x_{ij} \leq s + 100 \qquad j = m \qquad (5)$$

$$75\sum_{i\in L_j} x_{ij} + 75\sum_{i\in M_j} x_{ij} + 50z_j^{ML} + 75 \leq s + 75 \qquad \forall j \in W\setminus\{m\} \qquad (6)$$

$$75\sum_{i\in L_j} x_{ij} + 75\sum_{i\in M_j} x_{ij} \leq s + 75 \qquad j = m \qquad (7)$$

Some more constraints to model the z-variables... (8 - 31)

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in E$$

Tactical Planning: Runway Scheduling

problem description

- given
  - set of aircraft with different weight classes
  - earliest, schedule and latest times for each aircraft
  - minimum separation times between two aircraft types

source: wikipedia

- task
  - schedule aircraft as close as possible to their schedule times
  - penalize if assigned time is later than latest time
  - fair schedules

Tactical Planning: Runway Scheduling

1-Matching with Side Constraints: (Dyer/Wolsey 1990)

$$\min \sum_{i=1}^{n} \sum_{j \in T_i} c_{ij} \cdot x_{i,j}$$

subject to

each aircraft has to be scheduled

each slot can be used at most once

minimum separation time

Tactical Planning: Runway Scheduling

1-Matching with Side Constraints: (Dyer/Wolsey 1990)

$$\min \quad \sum_{i=1}^{n} \sum_{j \in T_i} c_{ij} \cdot x_{i,j}$$

subject to

$$\sum_{j \in T_i} x_{i,j} = 1 \qquad \forall i \in \{1, \ldots, m\}$$

$$\sum_{i=1}^{n} x_{i,j} \leq 1 \qquad \forall j \in T$$

$$x_{i,j} + \sum_{l=j+1}^{j+\lceil \frac{\delta_{i,k}}{\Delta t} \rceil} x_{k,l} \leq 1 \qquad \forall i \in \{1, \ldots, n\}, \forall j \in T_i, \forall k \neq i$$

$$x_{i,j} \in \{0, 1\}$$

Precendence Constraints on Runway: Structural Investigation

$$\sum_{j \in W} x_{i,j} = 1, \qquad \forall i \in A,$$

$$\sum_{i \in A} x_{i,j} \leq 1, \qquad \forall j \in W,$$

$$\sum_{j=1}^{a} x_{k_1,j} \geq \sum_{j=1}^{a} x_{k_2,j}, \qquad \forall a \in W \setminus \{\max(W)\}, (k_1, k_2) \in \text{Prec},$$

$$x_{i,j} \in \{0,1\}, \qquad \forall i \in A, j \in W.$$

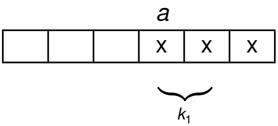
	a				
k_2	k_1				

One Precedence Constraint in Bipartite Matching

- poly-time problem if precedence constraint graph is series-parallel (Lawler 1978)
- In the general case bipartite matching with additional precedence constraints is NP-hard
- First consider one precedence constraint only, assume $|A| = |W| = n$
  - constraints remain feasible for several precendences

Facets for Bipartite Matching with one Precendence

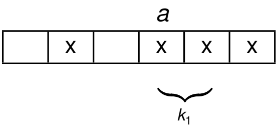
On the last $n - a + 1$ slots:
Forbid placing k_1 together with $n - a$ aircraft occupying all slots behind a with aircraft different from $\{k_1, k_2\}$.



$|F| = 2, \sum \leq 2.$

Facets for Bipartite Matching with one Precendence

On the last $n - a + 1$ slots and on y slots before a:
Forbid placing k_1 together with $n - a + y$ aircraft occupying all slots behind a and y before a with aircraft different from $\{k_1, k_2\}$.

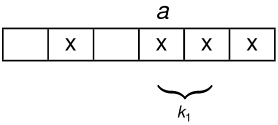


$|F| = 3, \sum \leq 3.$

Facets for Bipartite Matching with one Precendence

Example: $F = \{1,2,3\}, a = 4, y = 1, S_1 = \{2\}, S_2 = \{4,5,6\}$

$$\begin{aligned} & x_{1,2} + x_{1,4} + x_{1,5} + x_{1,6} \\ & + x_{2,2} + x_{2,4} + x_{2,5} + x_{2,6} \\ & + x_{3,2} + x_{3,4} + x_{3,5} + x_{3,6} \\ & + x_{k_1,4} + x_{k_1,5} \leq 3 \end{aligned}$$



$|F| = 3, \sum \leq 3.$

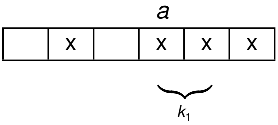
Facets for Bipartite Matching with one Precendence

Lemma

Let $a \in \{3, \ldots, n-1\}$, $y \in \{0, \ldots, a-3\}$, $F \subset A \setminus \{k_1, k_2\}$ $|F| = n - a + y$,
$S_1 \in \mathcal{P}(\{1, \ldots, a-2\} \setminus \{2\}) \cup \mathcal{P}(\{2, \ldots, a-2)$ with $|S_1| = y$, and
$S_2 = \{a, \ldots, n\}, S_1, S_2 \subset W$
Then, the inequalities

$$\sum_{i \in F} \sum_{j \in S_1} x_{i,j} + \sum_{j \in S_2 \setminus \{n\}} x_{k_1,j} + \sum_{i \in F} \sum_{j \in S_2} x_{i,j} \leq n - a + y$$

define a facet of Matching & One Precedence.



$|F| = 3$, $\sum \leq 3$.

Facets for Bipartite Matching with one Precendence

Lemma

Let $a \in \{3,\dots,n-1\}$, $y = 0$, if $a = n-1$ and $y \in \{0,\dots,a-3\}$ otherwise, $z \in \{1,\dots,n-a\}$, $F \subset A \backslash \{k_1, k_2\}$, with $|F| = n-a+y-z$, $S_1 \in \mathcal{P}(\{1,\dots,a-2\} \backslash \{2\}) \cup \mathcal{P}(\{2,\dots,a-2\})$ with $|S_1| = y$, $S_3 \in \mathcal{P}(\{a+1,\dots,n\} \backslash \{n-1\}) \cup \mathcal{P}(\{a+1,\dots,n-1\})$ with $|S_3| = z$, and $S_2 = \{a,\dots,n\} \backslash S_3$. Then the inequalities

$$\sum_{i \in F} \sum_{j \in S_1} x_{i,j} + \sum_{j \in S_2 \cup S_3 \backslash \{n\}} x_{k_1,j} + \sum_{i \in F} \sum_{j \in S_2} x_{i,j} + \sum_{i \in \tilde{A}} \sum_{j \in S_3} x_{i,j} \leq n-a+y$$

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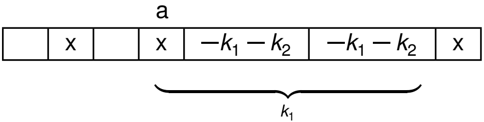

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Let $a \in \{3, \ldots, n-2\}$, $b \in \{a+1, \ldots, n-1\}$, $y = 0$ if $b = a+1$, $y \in \{0, \ldots, a-3\}$, $F \subset A \setminus \{k_1, k_2\}$, with $|F| = b - a + y - 1$, $S_1 \in \mathcal{P}(\{1, \ldots, a-2\} \setminus \{2\}) \cup \mathcal{P}(\{2, \ldots, a-2\})$ with $|S_1| = y$, $S_2 = \{a, \ldots, b-1\}$, and $S_3 = \{b, \ldots, n\}$. Then the inequalities

$$\sum_{i \in F} \sum_{j \in S_1} x_{i,j} + \sum_{j \in S_2} x_{k_1,j} + \sum_{i \in F} \sum_{j \in S_2} x_{i,j} + \sum_{j \in S_3 \setminus \{n\}} 2x_{k_1,j} + \sum_{i \in \tilde{A}} \sum_{j \in S_3} x_{i,j} \leq n - a + y$$

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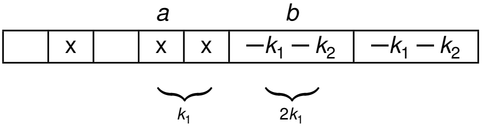

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Outline

- pre-tactical and tactical planning planning: time-window assignment and runway scheduling
- for both planning phases: affect of uncertainties, and
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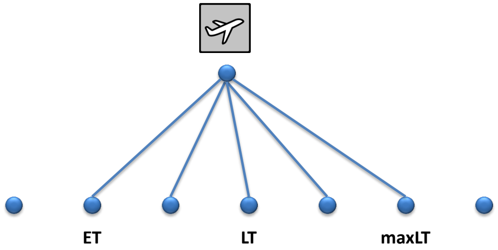
Uncertain Parameters

- disturbances affect ET, LT and maxLT of an aircraft

⇒ each realization yields an interval [ET, maxLT] of feasible assignments:



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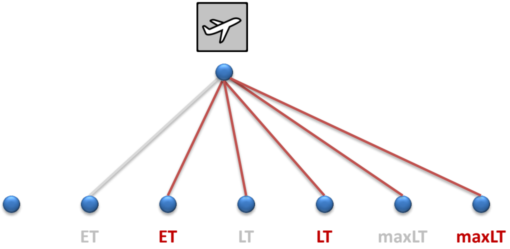

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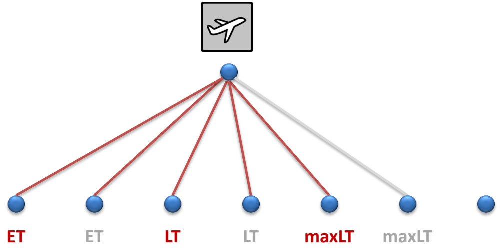

Impact of Uncertainties

- time windows of 10 minutes
- random disturbances (Gauss distribution, $\mu = 1, \sigma = 1.5$)

Number of Aircraft	Time Horizon (hrs)	Runtime (sec)	Objective Value	Delayed Aircraft (%)	Infeasible Assignments (%)
100	2.5	0.43	48.00	33.60	23.80
200	5	2.41	53.40	25.80	25.50
400	10	$>$170.86[1]	149.33[1]	34.83[1]	27.17[1]

- most runtimes are very low
  $\Rightarrow$ approaches can be used in practice
- about 20% of the aircraft are assigned to infeasible time windows
  $\Rightarrow$ enrich approaches by protection against uncertainties is crucial

[1] 40% of the instances exceeded time limit (15 min); averages taken over the 60% that could be solved to optimality

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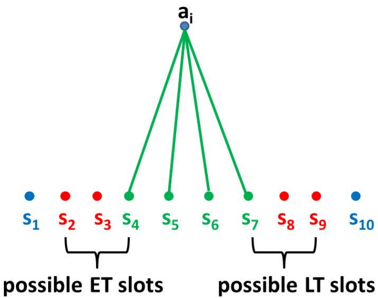
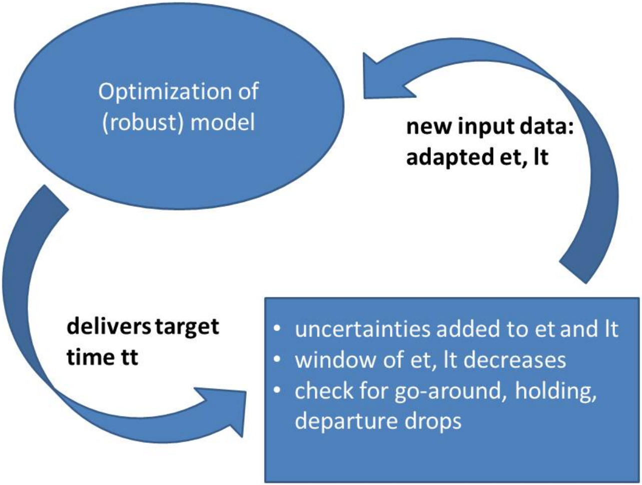
Strict Robustness



- **strict robustness**
  ⇒ delete all uncertain arcs

Simulation


Optimization of (robust) model
new input data: adapted et, lt
delivers target time tt
• uncertainties added to et and lt
• window of et, lt decreases
• check for go-around, holding, departure drops

Test Runs

- 3 scenarios
- 20 runs each
- 50 different randomly chosen aircraft per run
- high/med traffic (50 acft in 50mins/ 50acft 80 mins)
- high/low uncertainty

Scenario 1: High Traffic Demand, Low Uncertainty

Table : Results scenario 1: RTIM vs. TIM (slot size 75s)

Measurements	**RTIM**	**TIM**	**TIM - RTIM**
GoAround	0.2	1.8	1.6
Dep. drop	0.15	0.4	0.25
Makespan [s]	3064	3144	80
			TIM/RTIM
Changed Pos / SimStep	0.29	0.69	2.38
Changed TT / acft [min]	1.95	3.24	1.67
Obj. func. value / acft [s]	276.5	359	1.29
Comp. runtime [s]	12.5	13.1	1.05

Scenario 2: High Traffic Demand, High Uncertainty

Table : Results scenario 2: RTIM vs. TIM (slot size 75s)

Measurements	RTIM	TIM	TIM - RTIM
GoAround	0	2.4	2.4
Dep. drop	0	0.9	0.9
Makespan [s]	3269	3274	5
			TIM/RTIM
Changed Pos / SimStep	1.26	3.13	2.48
Changed TT / acft [min]	6.67	8.56	1.28
Obj. func. value / acft [s]	484.7	484.6	1.0
Comp. runtime [s]	31.8	26.4	0.83

Consequences

...we found

- stable plans: less replannings
- less go-arounds
- (strict) robustification is not costly
- and can be computed very fast

Less Conservative Concept: Recoverable Robustness

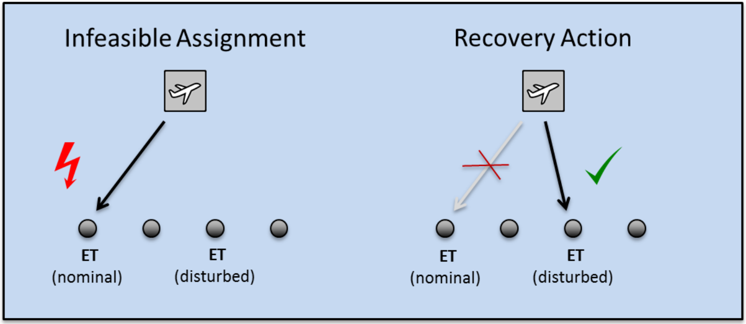
- consider nominal solutions (instead of strict robust solutions)
  → may become infeasible by disturbances
- ensure that feasibility can be "recovered" with minimal effort
  (by a certain recovery action)

→ developed for timetabling in railways

→ not found in ATM context yet

Recoverable Robustness

...in our application (taking it as *b*-matching):



$x_{ij} = 1$, j infeasible in scenario k $\Rightarrow$ 2nd-stage-assignment: $y^k_{il} = 1$

Recoverable Robustness

objective

minimize delay costs of nominal solution
+ worst case costs for recovery action

recovery action

determine feasible assignment "as close as possible" to current assignment

→ costs in scenario k:

minimum (squared) distances of nominal assigned time windows
to time windows feasible in scenario k

Recoverable Robustness

$$\min_x \sum_{i\in A}\sum_{j\in W_i} c_{ij}x_{ij} + \max_{k\in K}\min_{y^k} \sum_{i\in A}\left(\sum_{j\in W_i} j\cdot x_{ij} - \sum_{l\in W_i^k} l\cdot y_{il}^k\right)^2$$

$$\begin{aligned}
\text{s.t.} \quad \sum_{j\in W_i} x_{ij} &= 1 && \forall i\in A\\
\sum_{i\in A_j} x_{ij} &\leq b && \forall j\in W\\
\sum_{j\in W_i^k} y_{ij}^k &= 1 && \forall i\in A,\ \forall k\in K\\
\sum_{i\in A_j^k} y_{ij}^k &\leq b && \forall j\in W,\ \forall k\in K\\
x_{ij},\ y_{ij}^k &\in \{0,1\}
\end{aligned}$$

x : first stage assignment (for nominal scenario)

y^k : second stage (recovery) assignment in scenario k

$W_i^{(k)}$: feasible time windows (in scenario k)

Recoverable Robustness - Simplifications

- consider linear recovery term

$$\min_{x} \sum_{i\in A}\sum_{j\in W_i} c_{ij}x_{ij} + \max_{k\in K}\min_{z^k} \sum_{i\in A}\sum_{j\in W_i^k} c_i^{rep} z_{ij}^k, \quad \text{with } z_{ij}^k = \begin{cases} 1, & x_{ij}=0 \text{ and } y_{ij}^k = 1 \\ 0, & \text{otherwise.} \end{cases}$$

- consider recovery to strict robust solution

$$\min_{x,y} \sum_{i\in A}\sum_{j\in W_i} c_{ij}x_{ij} + \sum_{i\in A}\left(\sum_{j\in W_i} j\cdot x_{ij} - \sum_{l\in W_i^k} l\cdot y_{il}\right)^2$$

- consider recovery to strict robust solution with linear recovery term

$$\min_{x,z} \sum_{i\in A}\sum_{j\in W_i} c_{ij}x_{ij} + \sum_{i\in A}\sum_{l\in W_i^k} c_i^{rep} z_{ij}, \quad \text{with } z_{ij} = \begin{cases} 1, & x_{ij}=0 \text{ and } y_{ij} = 1 \\ 0, & \text{otherwise.} \end{cases}$$

Recoverable Robustness - Simplifications

Linear recovery term:

$$\min_{x} \sum_{i \in A} \sum_{j \in W_i} c_{ij} x_{ij} + \max_{k \in K} \min_{z^k} \sum_{i \in A} \sum_{j \in W_i^k} c_i^{rep} z_{ij}^k, \quad \text{with } z_{ij}^k = \begin{cases} 1, & x_{ij} = 0 \text{ and } y_{ij}^k = 1 \\ 0, & \text{otherwise.} \end{cases}$$

- count replanned aircraft with general replanning cost factor (not considering distances between x- and y^k-assignments)

→ don't consider different weight classes (yields general b-Matching problem)

⇒ second-stage constraints:

totally unimodular
(x and k fixed)

→ relax integrality, dualize

→ min-max structure

$$\begin{aligned} \sum_{j \in W_i^k} y_{ij}^k &= 1 && \forall i \in A \\ \sum_{i \in A_j^k} y_{ij}^k &\leq b && \forall j \in W \\ y_{ij}^k - z_{ij}^k &\leq x_{ij} && \forall ij \in E^k \\ y_{ij}^k,\ z_{ij}^k &\in \{0,1\} && \forall ij \in E^k \end{aligned}$$

Recoverable Robustness - Simplifications

Recovery to strict robust solution with linear recovery term:

$$\min_{x,z} \sum_{i \in A} \sum_{j \in W_i} c_{ij} x_{ij} + \sum_{i \in A} \sum_{l \in W_i^k} c_i^{rep} z_{ij}, \quad \text{with } z_{ij} = \begin{cases} 1, & x_{ij} = 0 \text{ and } y_{ij} = 1 \\ 0, & \text{otherwise.} \end{cases}$$

- considers only nominal case and strict robust scenario
  - → only two assignment problems with linear objective function to solve
  - → no "min max min"-structure anymore
- requires feasibility of strict robust approach
- count replanned aircraft with general replanning cost factor (not considering distances between x- and y-assignments)
  - → no fairness assured

Recoverable Robustness - Simplifications

Recovery to strict robust solution:

$$\min_{x,y} \sum_{i\in A}\sum_{j\in W_i} c_{ij}x_{ij} + \sum_{i\in A}\left(\sum_{j\in W_i} j\cdot x_{ij} - \sum_{l\in W_i^k} l\cdot y_{il}\right)^2$$

- considers only nominal case and strict robust scenario (i.e., for each aircraft take smallest possible time window)
  - → no "min max min"-structure anymore
  - → assignment problems with quadratic objective function to solve
- requires feasibility of strict robust approach
- algorithmically: Reformulation-Linearization Technique RLT
  - Balas, Ceria, Cornuéjols (1993)
  - Lovász, Schrijver (1991)
  - Sherali, Adams (1990)

Recoverable Robustness - Simplifications

Recovery to strict robust solution - RLT

$$
\begin{aligned}
\min_{x} \quad & \sum_{i\in A}\sum_{j\in W_i} c_{ij}x_{ij} + \sum_{i\in A}\left(\sum_{j\in W_i} j\cdot x_{ij} - \sum_{l\in W_i^R} l\cdot y_{il}\right)^2 \\
\text{s.t.} \quad & \sum_{j\in W_i} x_{ij} = 1 && \forall i\in A \\
& \sum_{i\in A_j} x_{ij} \leq b && \forall j\in W \\
& \sum_{l\in W_i^R} y_{il} = 1 && \forall i\in A \\
& \sum_{i\in A_l^R} y_{il} \leq b && \forall l\in W \\
& x_{ij},\, y_{il} \in \{0,1\}
\end{aligned}
$$

Recoverable Robustness - Simplifications

Recovery to strict robust solution - Reformulation & Linearization:

$$\min_x \sum_{i\in A}\sum_{j\in W_i} c_{ij}x_{ij} + \sum_{i\in A}\left(\left(\sum_{j\in W_i} j\cdot x_{ij}\right)^2 + \left(\sum_{l\in W_i^R} l\cdot y_{il}\right)^2 - 2\sum_{j\in W_i}\sum_{l\in W_i^R} jl\cdot x_{ij}y_{il}\right)$$

$$\text{s.t.} \quad \sum_{j\in W_i} x_{ij} = 1 \quad \forall i\in A$$

$$\sum_{i\in A_j} x_{ij} \leq b \quad \forall j\in W$$

$$\sum_{l\in W_i^R} y_{il} = 1 \quad \forall i\in A$$

$$\sum_{i\in A_l^R} y_{il} \leq b \quad \forall l\in W$$

$$x_{ij},\ y_{il} \in \{0,1\}$$

Recoverable Robustness - Simplifications

Recovery to strict robust solution - RLT

$$\min_x \sum_{i\in A}\sum_{j\in W_i} c_{ij}x_{ij} + \sum_{i\in A}\left(\sum_{j\in W_i} j^2\cdot x_{ij} + \sum_{l\in W_i^R} l^2\cdot y_{il} - \sum_{j\in W_i}\sum_{l\in W_i^R} 2jl\cdot \underbrace{x_{ij}y_{il}}_{=z_{ijl}}\right)$$

$$\text{s.t.} \quad \sum_{j\in W_i} x_{ij} = 1 \quad |\cdot y_{il} \quad \forall l\in W_i^R \qquad \forall i\in A$$

$$\sum_{i\in A_j} x_{ij} \le b \qquad \forall j\in W$$

$$\sum_{l\in W_i^R} y_{il} = 1 \quad |\cdot x_{ij} \quad \forall j\in W_i \qquad \forall i\in A$$

$$\sum_{i\in A_l^R} y_{il} \le b \qquad \forall l\in W$$

$$x_{ij},\ y_{il} \in \{0,1\}$$

Recoverable Robustness - Simplifications

Recovery to strict robust solution - RLT

$$\min_x \sum_{i\in A}\sum_{j\in W_i} c_{ij}x_{ij} + \sum_{i\in A}\left(\sum_{j\in W_i} j^2\cdot x_{ij} + \sum_{l\in W_i^R} l^2\cdot y_{il} - \sum_{j\in W_i}\sum_{l\in W_i^R} 2jl\cdot z_{ijl}\right)$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{j\in W_i} x_{ij} = 1 && \forall i\in A\\
& \sum_{j\in W_i} z_{ijl} = y_{il} && \forall l\in W_i^R,\ \forall i\in A\\
& \sum_{i\in A_j} x_{ij} \leq b && \forall j\in W\\
& \sum_{l\in W_i^R} z_{ijl} = x_{ij} && \forall j\in W_i,\ \forall i\in A\\
& \sum_{i\in A_l^R} y_{il} \leq b && \forall l\in W\\
& x_{ij},\ y_{il} \in \{0,1\},\quad z_{ijl} \geq 0
\end{aligned}$$

Computational Results

Approach	Runtime	ObjVal (delay)	delayed Acft	infeasible Ass.	replanned Acft	max replan dist.	mean replan dist.	quadratic recovery term
nominal	2.95	53.4	51.6	51.2	-	-	-	-
strict robust	4.28	546.4	200*	10.4	-	-	-	-
recovery to strict quadratic	85.98	274.8	159.4	20.0	115.4	2.0	0.60	129.8
recovery to strict linear	5.95	193.6	117.6	28.6	88.2	8.8	1.11	963.2
recovery to strict linear, restricted	28.88	199.2	117.4	28.0	113.2	2.0	0.97	356.2

Tested 5 instances: 200 acft on 10min-windows, normally distributed disturbances ("restricted": max replan dist. ≤ 2)

Uncertainty set: $\mu \pm k \cdot \sigma$ ($\mu = 1$, $\sigma = 1.5$, $k = 1$)   * scheduled time window not contained in chosen uncertainty set

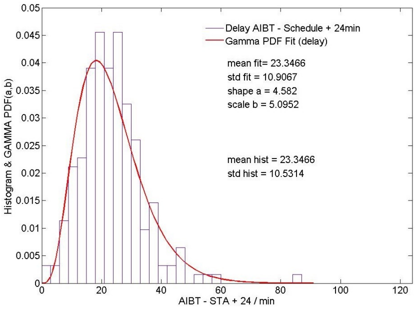
- recoverable approaches: ObjVal / infeasible Ass. between nominal and strict robust (closer to nominal)
- quadratic recovery: least infeasible Ass.
- linear recovery: low runtime, little replanned aircraft, but rather unfair (max replan dist./quadratic recovery term)
- restricted linear recovery: still not as fair as quadratic (quadratic recovery term)

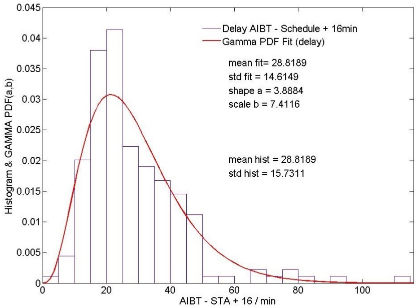
Further Approaches for Protection Against Uncertainties

- (two-stage) stochastic approach
- mixed robust-stochastic model, in which protection against uncertainties can be tuned according to needs

Modeling of Arrival Delay / Statistical Analysis of Empirical Data

- we analyzed empirical delay data from a large German airport
- we applied a Γ-distribution model to the delay statistics of a single day (all flights) as well as a 6-month period (single flights):





Summary

- mathematical approaches for pre-tactical and tactical planning in ATM
- yields *b*-matching problem (plus further constraints)
- polyhedral description of bipartite matching with 1 precendence
- protection against uncertainties with (recoverable) robust optimization, one step and within simulation
- analysis of delay statistics

Conclusions

- uncertainties in input occur in many practical applications
- they can be treated already in the mathematical model
- “Often” the resulting optimization problems are “not much more” difficult.

Thank you for your attention!