# KWIM-Festtage

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Konstanz Women in Mathematics

## Cora Sadosky, a collaborator and a friend: a survey of joint work on scattering systems, function theory on the polydisc, and related hermitian sos decompositions

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**Abstract.** Cora Sadosky (Buenos Aires, 1940 – Long Beach, California, 2010) made important contributions to harmonic analysis and operator theory. She was also an outstanding woman, a firm champion of human rights (something that cost her dearly during the years of the military rule in Argentina), and deeply committed to the advancement of women in mathematics (she was the president of AWM during the years 1993–1995, and in 2012 AWM established the AWM–Sadosky Research Prize in Analysis in her honor). I was privileged to be her friend since the early 1990s, and somewhat later (since around 2000), her collaborator.

In this talk I will say a few words about Cora, her life and work (for much more, see the lovely collection [12]), and our friendship. I will then discuss our joint work (joint also with Joe Ball, and later with Dima Kaliuzhnyi-Verbovetskyi) [4, 5, 3] on function theory and operator theory related to the the unit polydisc

$$\mathbb{D}^d = \left\{ (z_1, \ldots, z_d) \in \mathbb{C}^d \colon |z_1|, \ldots, |z_d| < 1 \right\}.$$

The main idea is the representation of a contractive analytic function on the unit polysisc as *the scattering function of a scattering system with d evolution operators*. This idea originates implicitly with Livšic [11, 6] and explicitly with Lax–Phillips [10] and Adamyan–Arov [1] in the case $d = 1$, where it became one of the cornerstones of a deep interaction between complex analysis, operator theory, system theory, and mathematical physics (see, e.g., [9]). It was probably first considered by Cotlar–Sadosky [7, 13] for general $d$.

The Konstanz group in real algebraic geometry is the leader in the study of multidimensional moment problems and of sums of squares (sos) decompositions and Positivstellensätze. Therefore it will be only appropriate to finish my talk by showing how our work relates to certain hermitian sos decompositions in 2 complex variables (called Agler decompositions after Agler's pioneering work [2] that initiated the modern operator theoretic study of the polydisc), and to the 2-dimensional trigonometric moment problem as studied by Geronimo–Woerdemann [8]. Time permitting, I will also mention some ongoing and future work.

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