

Moment Problem: Schmüdgen Theorem (1991)

María López Quijorna

MASTER SEMINAR ON REAL ALGEBRAIC GEOMETRY
UNIVERSITY OF KONSTANZ

Abstract

Let T be a finitely generated preordering of $\mathbb{R}[X_1, \dots, X_d]$ such that the associated basic closed semialgebraic set K is compact. Schmüdgen Theorem (1991) asserts that in this case every linear functional $L : \mathbb{R}[X_1, \dots, X_d] \rightarrow \mathbb{R}$ which satisfies $L(T) \subseteq \mathbb{R}_{\geq 0}$ can be represented as an integral with respect to a positive Borel measure supported in K , i.e. there exists a positive Borel measure μ on K such that $L(f) = \int f d\mu$ for all $f \in \mathbb{R}[X_1, \dots, X_d]$. This theorem supposed a positive result for the Multidimensional Moment Problem in the compact case. In this seminar we will review the original proof by Schmüdgen (1991) which makes use of the Krivine-Positivstellensatz (1964), the Gelfand-Naimark-Segal Construction (1943) and the Spectral Theorem for Bounded Operators.