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POSITIVE POLYNOMIALS AND MOMENT PROBLEMS–SS 2019

Recap Sheet 1

*This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Wednesdays 2-3 pm in room F408.*

Denote by $\mathbb{R}[\underline{X}] = \mathbb{R}[X_1, \ldots, X_n]$ the ring of polynomials in n variables with real coefficients.

1) Recall Hilbert's 1888 Theorem about the relation of $\mathcal{P}_{n,d}$ and $\Sigma_{n,d}$. Give examples of polynomials $p \in \mathcal{P}_{n,d} \backslash \Sigma_{n,d}$ for the cases $(n,d) = (3,6)$ and $(n,d) = (4,4)$.

2) State the Stengle-Krivine Positivstellensatz and recall the structure of the proof form your real algebraic geometry script. Which principle in real algebraic geometry plays a crucial role in the proof?

3) State Hilbert's 17th problem and solve it using the Stengle-Krivine Positivstellensatz.

4) Recall the definition of saturated preordering.

5) Let $n = 1$. Give an example of a compact basic closed semi-algebraic set K and two descriptions of K such that for one the corresponding preordering is saturated while for the other is not.

6) Let $n = 1$. Recall the natural description of a basic closed semi-algebraic set in $\mathbb{R}$. Use this to give an example of a non-compact basic closed semi-algebraic set K and two descriptions of K such that for one the corresponding preordering is saturated while for the other is not.

7) Let $n = 2$. Give an example of a compact basic closed semi-algebraic set K and two descriptions of K such that for one the corresponding preordering is saturated while for the other is not.

8) Let $n = 2$ and $S := \{X^3, 1 - X\}$. Is the preordering T_S saturated? If not, can you give another description of K_S such that the corresponding preordering is saturated?

9) What can you say about saturation if $n \geq 3$?

10) Give an example of a saturated preordering in $\mathbb{R}[X_1, X_2, X_3]$.