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POSITIVE POLYNOMIALS AND MOMENT PROBLEMS–SS 2019

Recap Sheet 2

This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** *need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Wednesdays 2-3 pm in room F408.*

1) Recall the definition of a character space of a real algebra and the notion of Gelfand transform.

2) Describe the character space of the polynomial algebra $\mathbb{R}[X_1, \dots, X_n]$ and the Gelfand transform of $f \in \mathbb{R}[X_1, \dots, X_n]$.

3) Recall the definition of Archimedean T–module and characterise Archimedean quadratic modules in $\mathbb{R}[X_1, \dots, X_n]$.

4) State the general version of the Representation Theorem given in the class. Which well-known results can be derived form this theorem?

5) State the Schmüdgen Positivstellensatz and show that the compactness assumption is essential.

6) Recall the definition of first and second dual cone in a TVS. Do you know any characterisation of the second dual cone?

7) How do the Nichtnegativstellensätze relate to closures of $2d$–power modules?

8) Recall the definitions of topological algebra and Banach algebra. Give concrete examples for both notions.

9) Give examples of lc and lmc topologies on $\mathbb{R}[X_1, \dots, X_n]$ and describe the closure of the quadratic module $\sum \mathbb{R}[X_1, \dots, X_n]^2$ w.r.t. these topologies.

10) What can you say about the closure of Archimedean quadratic modules in a unital commutative $\mathbb{R}$–algebra w.r.t. lmc topologies?