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POSITIVE POLYNOMIALS AND MOMENT PROBLEMS–SS 2019

Recap Sheet 4

This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** *need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Wednesdays 2-3 pm in room F408.*

Denote by $\mathbb{R}[\underline{X}] := \mathbb{R}[X_1, \dots, X_n]$ the ring of polynomials in n variables with real coefficients.

1) State the classical Hamburger, Stieltjes and Hausdorff moment problems and sketch their proofs.

2) Formulate a solution to the $[-1, 1]$–moment problem.

3) Let $m := (m_j)_{j\in\mathbb{N}} \subseteq \mathbb{R}^{\mathbb{N}_0}$ be a sequence of real numbers and set $g := 1 \in \mathbb{R}[X]$. In the lecture we defined the sequence $g(E)m$. How is the map $g(E) : \mathbb{R}^{\mathbb{N}_0} \to \mathbb{R}^{\mathbb{N}_0}$ defined?

4) Define the moment matrix associated to a linear functional from $\mathbb{R}[\underline{X}] \to \mathbb{R}$. Recall necessary and sufficient conditions for a matrix to be psd from your linear algebra course.

5) Give as many necessary and sufficient conditions as you can for a linear functional from $\mathbb{R}[\underline{X}] \to \mathbb{R}$ to be non-negative on a quadratic module of $\mathbb{R}[\underline{X}]$.

6) How can the closure results in Section 1 be used to solve the moment problem?

7) Recall the definition of *determinate* Radon measure on $\mathbb{R}^n$.

8) Let $K \subseteq \mathbb{R}^n$ be compact. How many K–representing Radon measures for a linear functional $\mathbb{R}[\underline{X}] \to \mathbb{R}$ do exist?

9) Give a sufficient condition for a linear functional $\mathbb{R}[\underline{X}] \to \mathbb{R}$ to have a unique representing measure.