Universität Konstanz
Fachbereich Mathematik und Statistik
Dr. Maria Infusino
Patrick Michalski

POSITIVE POLYNOMIALS AND MOMENT PROBLEMS–SS 2019

Recap Sheet 5

This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** *need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Wednesdays 2-3 pm in room F408.*

1) Recall the definition of adjoint of an operator on a Hilbert space for both the the bounded and the unbounded case.

2) State the spectral theorem for a tuple of self-adjoint bounded (resp. unbounded) operators on a Hilbert space.

3) State Schmüdgen's solution to the KMP for K compact basic closed semi-algebraic subset of $\mathbb{R}^n$.

4) How can we construct a Hilbert space out of a linear functional non-negative on the squares of polynomials in $\mathbb{R}[X_1, \ldots, X_n]$?

5) To what aim is the Stengle Positivstellensatz used in the original proof of Schmüdgen's solution to the KMP for K compact basic closed semi-algebraic subset of $\mathbb{R}^n$?

6) Compare the operator theoretical proofs of Schmüdgen's and Putinar's theorems for the KMP by identifying similarities and differences.

7) State Nussbaum's solution to the KMP for $K = \mathbb{R}^n$.

8) Compare the proofs of Schmüdgen's and Nussbaum's theorems by identifying the main steps and differences.

9) What is the role of the multivariate Carleman condition in the proof of Nussbaum's theorem for the KMP for $K = \mathbb{R}^n$.

10) Do you know a version of Nussbaum's theorem for the KMP with K (not necessarily compact) basic closed semi-algebraic subset of $\mathbb{R}^n$?