Theorems of Pólya and Reznick

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Abstract

Let $f \in \mathbb{R}[\underline{X}] := \mathbb{R}[X_1, \ldots, X_n]$ be a homogeneous polynomial. Pólya's Theorem states that if f is positive on $\mathbb{R}^n_{\geq 0} \backslash \{\underline{0}\}$, then there exists $k \in \mathbb{N}_0$ such that $(\sum X_i)^k f$ has non-negative coefficients. The proof we give is due to Wörmann and uses the Representation Theorem for Archimedean T-modules. Modifiying this proof it is possible to obtain a similar result of Reznick, which allows a 'uniform choice of denominators' in Hilbert's 17th Problem. Moreover, we slightly generalize the statement of Reznick's Theorem.