



PROSEMINAR ON LINEAR ALGEBRA–WS 2016–2017

List of possible topics

This is just a list of possible topics where you can find some ideas and references for preparing your seminars. Please, feel free to search for other sources and/or to propose us a completely different topic in linear algebra on which you would like to build your own seminar.

1. Operators on inner product spaces

- (a) **Forms on inner product spaces and their representations**
(See e.g. [5, Section 9.2, pp.320–324])
- (b) **Nonnegative and positive forms on inner product spaces**
(See e.g. [5, Section 9.3, pp.325–329])
- (c) **Relation between positive operators, positive forms and positive matrices**
(See e.g. [5, Section 9.3, pp.329–331])
- (d) **Forms and projections of vectors spaces**
(See e.g. [5, Section 9.4, pp.332–334])

2. Applications of the spectral theorem

- (a) **Functions of diagonalizable normal operators**
(See e.g. [5, Section 9.5, pp.337–340])
- (b) **Applications involving nonnegative operators**
(See e.g. [5, Section 9.5, pp.340–343])
- (c) **Simultaneous diagonalization of families of normal operators**
(See e.g. [5, Section 9.5, pp.343–347])
- (d) **Spectral theorem and primary decomposition**
(See e.g. [5, Section 9.6, pp.349–352])
- (e) **Properties of normal operators on real vector spaces**
(See e.g. [5, Section 9.6, pp.352–354])
- (f) **Unitarily equivalence of normal operators**
(See e.g. [5, Section 9.6, pp.354–358])

3. Bilinear forms

- (a) **Bilinear forms on finite-dimensional vector spaces**
(See e.g. [5, Section 10.1, pp.360–366])
- (b) **Symmetric bilinear forms and their diagonalization**
(See e.g. [5, Section 10.2, pp.367–372])
- (c) **Skew-symmetric bilinear forms**
(See e.g. [5, Section 10.3, pp.375–379])
- (d) **Group preserving bilinear forms**
(See e.g. [5, Section 10.3, pp.379–383])

4. Direct methods for linear systems and their applications

(a) **QR Factorization Method**

(See e.g. [1, Section 6.4, pp.116–124] and [2, Section 14.3, pp.287–291])

(b) **Householder Algorithm**

(See e.g. [1, Section 7.3.4, pp.136–142])

(c) **Method of orthogonal vectors**

(See e.g. [3, Chapter 5, pp.125–135])

5. Linear algebraic problems in mathematical physics*

Topics connecting linear algebra and mathematical physics can be found for example in [8]. In particular, three mathematical problems appearing in quantum mechanical models are sketched in [8, Volume I, Chapter VIII, Section 11]: self-adjoint extensions of symmetric operators [8, Volume II, Chapter X]; spectral analysis [8, Volume IV, Chapter XII, Chapter XIII]; scattering theory [8, Volume III, Chapter XI].

6. Applications of Jordan forms to systems of linear ordinary differential equations*

(See e.g. [7, (pp. 39– 50)])

***NB.** *Topics in the area 5 and 6 can involve more advanced mathematical tools than the ones you have encountered in your studies so far . If you are interested to go inside one of these areas, please have a look at the suggested references and come to the preliminary meeting or contact us to fix an appointment. We can try find out together which specific topic in those areas might be more suitable to your own background.*

References

- [1] G. Allaire, S. M. Kaber *Numerical linear algebra*, Texts in Applied Mathematics, 55. Springer, New York, 2008.
- [2] W. Ford *Numerical linear algebra with applications*, Elsevier/Academic Press, Amsterdam, 2015.
- [3] L. Fox, *Introduction to numerical linear algebra*, Monographs on Numerical Analysis Clarendon Press, Oxford, 1964.
- [4] P. R. Halmos, *Finite-dimensional vector spaces*. Reprinting of the 1958 second edition. Undergraduate Texts in Mathematics. Springer-Verlag, New York-Heidelberg, 1974.
- [5] K. Hoffman, R. Kunze *Linear Algebra*, Prentice-Hall, Inc., Englewood Cliffs, N.J. 1971.
- [6] F. Lorenz, *Lineare algebra*, vol. I and II, Spektrum Akademischer Verlag, 2008.
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