

Positive polynomials and sum of squares in formal power series rings

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Abstract

Let $\mathbb{R}[[\underline{X}]] = \mathbb{R}[[X_1, \dots, X_n]]$ be the ring of formal power series and let $f \in \mathbb{R}[[\underline{X}]]$ be positive at every ordering of the Laurent series ring $\mathbb{R}((\underline{X}))$. We want to examine the question of whether f is a sum of squares. Using some basic characteristics of the power series rings, it is quite easy to prove that this holds when $n = 1$ and it does not hold when $n \geq 3$. In the case $n = 2$ the result also holds but the proof requires more advanced tools.