Universität Konstanz
Fachbereich Mathematik und Statistik
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TOPOLOGICAL ALGEBRAS–SS2018

Exercise Sheet 6

This exercise sheet aims to assess your progress and to explicitly work out more details of some of the results proposed in the previous lectures. Please, hand in your solutions in postbox 16 near F411 by Wednesday the 11th of July at 15:15. The solutions to this assignment will be discussed in the tutorial on Friday 13th of July (10:00–11:30 in F420).

1) *(5 points)* Let (X, τ) be an lb Hausdorff TA. Show that if (X, τ) has jointly continuous multiplication, then (X, τ) is α–normable.

2) *(5 points)* Let $\{(E_\alpha, \tau_\alpha), f_{\alpha,\beta}, I\}$ be a projective system of TAs and $J \subseteq I$ be a cofinal set. Show that the projective limit of the projective system $\{(E_\alpha, \tau_\alpha), f_{\alpha,\beta}, I\}$ is also a projective limit of the projective system $\{(E_\alpha, \tau_\alpha), f_{\alpha,\beta}, J\}$.

3) *(5 points)* Let $\{(E, \tau), f_\alpha, I\}$ be the projective limit of the projective system $\{(E_\alpha, \tau_\alpha), f_{\alpha,\beta}, I\}$ of Hausdorff TAs. Show that (E, τ) is a closed subalgebra of $(\prod_{\alpha \in I} E_\alpha, \tau_{\mathrm{prod}})$.

4) *(5 bonus points)* Show that the projective limit of a projective system of complete Hausdorff TAs is complete.

5) *(5 bonus points)* Show that the cartesian product of Hausdorff TAs endowed with the product topology is a Hausdorff TA.