



TOPOLOGICAL ALGEBRAS–SS 2018

Recap Sheet 1

*This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Thursdays 2-3 pm in room F408.*

- 1) Recall the definition of product topology τ_{prod} of a family $\{(X_i, \tau_i) : i \in I\}$ of topological spaces. Given a basis $\mathcal{B}_i$ of τ_i for $i \in I$ describe a basis of τ_{prod} .
- 2) Give an example of a function on the product of two topological spaces (endowed with the product topology) which is continuous but not jointly continuous.
- 3) Recall the definition of TA (resp. TA with continuous multiplication).
- 4) Give at least five concrete examples of TAs. Which ones are actually TAs with continuous multiplication?
- 5) List as much properties of the filter of neighbourhoods of the origin of a TA (resp. TA with continuous multiplication) as you can.
- 6) Give at least two conditions characterizing Hausdorff TA.
- 7) Is it possible to always topologically embed a TA into a unital TA? If yes, can this unital TA be constructed explicitly?
- 8) Recall the concepts of initial and final topology and give an example for each of them.
- 9) Let A be a TA and I an ideal of A . Show that the quotient map $\phi : A \rightarrow A/I$ is open when A/I is endowed with the quotient topology. Is this still true if A is only assumed to be a topological space (and I is just any subset of A)?
- 10) Let A be a normed algebra and I an ideal of A . Is A/I (endowed with the quotient topology) also a normed algebra?