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TOPOLOGICAL ALGEBRAS-SS 2018

Recap Sheet 2

This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Thursdays 2-3 pm in room F408.

- 1) Give some examples of m-sets of a K-algebra. Which operations preserve the multiplicativity of a set?
- 2) Recall the definition of m-barrel in a TA. Is an m-barrel in a TA always a neighbourhood of the origin? Justify your answer with a proof or a counterexample.
- 3) Recall the definition of lmc algebra and characterize lmc algebras in terms of m-barrels.
- 4) Examine the relation between Minkowski functionals and seminorms.
- 5) How is continuity of a seminorm related to the corresponding semiballs?
- 6) Characterize lmc algebras in terms of seminorms. How can a base of neighbourhoods of an lmc algebra be described using this characterization?
- 7) Establish an example of a TA which is not lmc.
- 8) How can one compare two lmc topologies on the same K-algebra?
- 9) Show that the maximum of a finite number of seminorms is itself a seminorm. Use this observation to obtain from a family of seminorms a directed family of seminorms which generates the same topology.
- 10) Recall the Hahn-Banach Theorem (introduced in TVS I, Theorem 5.1.2) as it is an important result in the theory of lc TVS.