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TOPOLOGICAL VECTOR SPACES II–WS 2019/2020

Bonus Sheet

This exercise sheet aims to assess your progress and to explicitly work out more details of some of the results proposed in the previous lectures. You may *hand in your solutions by Wednesday the 12th of February at 13:30 in order to score bonus points! Solutions to this assignment will be made available online on Maria's webpage.*

1) Given two sets X and Y, let E (resp. F) be the linear space of all functions from X (resp. Y) to $\mathbb{K}$ endowed with the usual addition and multiplication by scalars. For any $f \in E$ and $g \in F$, define:

$$\begin{array}{rrcl} f \otimes g: & X \times Y & \to & \mathbb{K} \\ & (x, y) & \mapsto & f(x)g(y). \end{array}$$

Show that $M := \text{span}\{f \otimes g : f \in E, g \in F\}$ is a tensor product of E and F.

Let E and F be two locally convex t.v.s. over $\mathbb{K}$. Denote by $E \otimes_\pi F$ the tensor product $E \otimes F$ endowed with the π–topology. Prove the following statements:

2) If $\mathcal{P}$ (resp. $\mathcal{Q}$) is a family of seminorms generating the topology on E (resp. on F), then the π–topology on $E \otimes F$ is generated by the family

$$\{p \otimes q : p \in \mathcal{P}, q \in \mathcal{Q}\},$$

where for any $p \in \mathcal{P}, q \in \mathcal{Q}, \theta \in E \otimes F$ we define:

$$(p \otimes q)(\theta) := \inf\{\rho > 0 : \theta \in \rho \, \text{conv}_b(U_p \otimes V_q)\}$$

with $U_p := \{x \in E : p(x) \leq 1\}$ and $V_q := \{y \in F : q(y) \leq 1\}$.

3) $E \otimes_\pi F$ is Hausdorff if and only if E and F are both Hausdorff.