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TOPOLOGICAL VECTOR SPACES II–WS 2019/2020

Exercise Sheet 5

This exercise sheet aims to assess your progress and to explicitly work out more details of some of the results proposed in the previous lectures. Please, hand in your solutions in postbox 18 near F411 by Wednesday the 22nd of January at 13:30. The solutions to this assignment will be discussed in the tutorial on Tuesday the 30th of January (13:30–15:00) in D406.

1) Let E be a locally convex metrizable t.v.s.. Prove that if E is not normable, then E cannot have a countable basis of bounded sets in E.

2) Let X be a set and $\mathcal{F}$ the space of real-valued functions on X endowed with the topology of pointwise convergence, that is, the projective topology on $\mathcal{F}$ w.r.t. the family $\{v_x : x \in X\}$, where $v_x \colon \mathcal{F} \to \mathbb{R}, f \mapsto f(x)$ for each $x \in X$.

Show that $\mathcal{F}$ is normable if and only if X is finite.

3) Let (X, τ) be a t.v.s. and $Y \subseteq X$ a linear subspace endowed with the subspace topology τ_Y induced by τ. Show that if $B \subseteq (Y, \tau_Y)$ is bounded, then $B \subseteq (X, \tau)$ is bounded.

Conclude that a bounded linear map from an LF-space into an arbitrary t.v.s. is always continuous.

4) Let E be an LF-space defined by the sequence $\{E_n\}_{n\in\mathbb{N}}$ such that $\dim(E_n) < \infty$ for all $n \in \mathbb{N}$. Prove the following statements:

a) If F is a normable t.v.s. and $u \colon F \to E$ a linear and continuous map, then $F/\ker(u)$ is finite dimensional, i.e. $\dim(F/\ker(u)) < \infty$.

b) If a linear subspace $M \subseteq E$ endowed with the subspace topology is normable, then $\dim(M) < \infty$.