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TOPOLOGICAL VECTOR SPACES II–WS 2019/2020
Interactive Sheet

Let X be a non-trivial vector space, $d : X \times X \to X$ a translation invariant metric and τ_d the topology induced by d.

Let us show together that (X, τ_d) is a metric space but not necessarily a t.v.s..

1) Show that the addition $a : X \times X \to X$ is τ_d-continuous.

2) Let us look at a counterexample showing that the scalar multiplication $m : \mathbb{K} \times X \to X$ is not necessarily τ_d-continuous.

   a) Let d be the discrete metric on X and suppose that the scalar multiplication is τ_d-continuous. Then for any $x \neq 0$ in X we have that $\frac{1}{n}x \to \ldots\ldots$ as $n \to \infty$,

   b) namely, for any U neighbourhood of the origin in (X, τ_d) we have that $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

   c) In particular, for $U = \{o\}$ we get $\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

   d) Then $x = 0$, which yields a contradiction.

Hence, for the discrete metric d on X the scalar multiplication is not τ_d-continuous and so (X, τ_d) is not a t.v.s..