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TOPOLOGICAL VECTOR SPACES II–WS 2019/2020

Interactive Sheet 3

Prove that: if E is a locally convex Hausdorff t.v.s with $E \neq \{o\}$, then for every $o \neq x_0 \in E$ there exists $x' \in E'$ s.t. $\langle x', x_0 \rangle \neq 0$, i.e. $E' \neq \{o\}$.

Proof. Let $o \neq x_0 \in E$.

a) Since (E, τ) is a locally convex Hausdorff t.v.s, we know that τ is generated by ....

.... and so there exists $p \in$ ....

b) Take $M := span\{x_0\}$ and define the $\ell : M \to \mathbb{K}$ by $\ell(\alpha x_0) := \alpha p(x_0)$ for all $\alpha \in \mathbb{K}$.

c) The functional ℓ is clearly .... and .... on M. Then by the Hahn-Banach theorem we have that there exists a linear functional $x' : E \to \mathbb{K}$ such that

....

d) Hence, $x' \in E'$ and $\langle x', x_0 \rangle =$ ....

$\square$