



## TOPOLOGICAL VECTOR SPACES II–WS 2019/2020

### Recap Sheet 1

*This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Thursdays 11:45–13:15 in room F408.*

- 1) Recall the definition of basis of a topology and of basis of a filter (introduced in TVS-I).
- 2) Give the definition of metrizable t.v.s. and characterize them using their neighbourhoods.
- 3) Give an example of a metric space  $(X, d)$  such that  $d$  is translation invariant and  $(X, d)$  is not a t.v.s.. Why is this an interesting example?
- 4) Recall the definition of a separating family of seminorms (introduced in TVS-I).
- 5) Do you know necessary and sufficient conditions for a t.v.s.  $X$  to be Hausdorff? What if  $X$  is assumed to be locally convex?
- 6) Do you know necessary and sufficient conditions for a locally convex t.v.s. to be metrizable?
- 7) Give the definition of Baire space.
- 8) Give an example of a t.v.s. which is not a Baire space.
- 9) Do you know any sufficient conditions which guarantee that a t.v.s. is a Baire space?
- 10) Give the definition of Fréchet space. List at least three examples of Fréchet spaces.