



TOPOLOGICAL VECTOR SPACES II–WS 2019/20

Recap Sheet 3

*This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Thursdays 11:45–13:15 in room F408.*

- 1) Recall the definition of accumulation point of a filter of a topological space. What is the relation between accumulation points and limit points of a filter of a t.v.s.?
- 2) Recall the definition of projective topology on a vector space. List at least two examples of projective topologies.
- 3) Give a criterion for a projective topology to be Hausdorff. Use this criterion to reprove the fact, that the product of Hausdorff spaces is itself a Hausdorff space.
- 4) Compare the concepts of inductive and projective topology.
- 5) Do you know a vector space which can be equipped with both an inductive and a projective topology? What are the defining sequences?
- 6) Recall the definition of projective limit and convince yourself that any product of l.c. t.v.s. can be realized as a projective limit.
- 7) Do you know a property that is satisfied by any projective limit?
- 8) Give a sufficient condition for a continuous linear map between two t.v.s. to be open.
- 9) State the closed graph theorem and sketch how to deduce it from the open mapping theorem.
- 10) Are the closed graph and the open mapping theorem equivalent? Justify or answer with a sketch of a proof or a counterexample!