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TOPOLOGICAL VECTOR SPACES II–WS 2019/20

Recap Sheet 4

This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** *need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Thursdays 11:45–13:15 in room F408.*

1) Give a criterion for a Hausdorff space to be compact in terms of accumulation points.

2) Recall the definitions of relatively compact and precompact sets. In particular, recall the concept of completion of a Hausdorff t.v.s. (introduced in TVS-I).

3) Recall the definition of bounded subsets of a t.v.s. and give some examples of classes of such sets.

4) Give an example of a basis of bounded subsets of the Schwartz space $\mathcal{S}(\mathbb{R}^d)$ for some $d \in \mathbb{N}$.

5) How are the terms "bounded", "closed" and "compact" related to each other when referred to a subset of a t.v.s.?

6) Provide an example of a t.v.s. where the Heine-Borel property fails to hold.

7) State and prove a characterization of boundedness for subsets of a t.v.s. in terms of sequences.

8) Give a criterion for a Hausdorff l.c. t.v.s. to be normable.

9) Recall the definition of (linear) bounded map and explain why continuous linear maps between t.v.s. are bounded. Does the converse hold? If yes, prove it. If not, do you know any class of t.v.s. in which this holds?

10) For a sequence S of points in a Hausdorff t.v.s. is the property of being Cauchy sufficient to conclude that S is compact, relatively compact or precompact? Justify your answers!