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TOPOLOGICAL VECTOR SPACES II–WS 2019/20

Recap Sheet 5

This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** *need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Thursdays 11:45–13:15 in room F408.*

1) Recall the definition of the algebraic and topological dual space of a t.v.s. and the notion of pairing between a space and its dual.

2) Recall the definition of polar of a subset of a t.v.s.. What can you say about the polar of a cone?

3) Prove that the polar of a bounded subset of a t.v.s. is absorbing. Why is this a crucial observation?

4) Recall the definition of Σ–topology on the topological dual of a t.v.s. and list at least three important examples.

5) Is a Σ–topology on the topological dual of a t.v.s. a locally convex topology? Justify your answer with a proof or give a counterexample.

6) Give a sufficient condition for a Σ–topology on the topological dual of a t.v.s. to be Hausdorff.

7) Let E be a t.v.s. and for $x \in E$ consider the map $v_x : E' \to \mathbb{K},\ x' \mapsto \langle x', x\rangle$. Give an example of a class of topologies on E' for which v_x is continuous for all $x \in E$.

8) When can a t.v.s. E be regarded as the topological dual of its weak dual E'_σ?

9) Is there a sufficient condition ensuring that the topological dual of a t.v.s. contains a non-zero element? Give an example of a t.v.s. whose topological dual is trivial.

10) State the Banach-Alaoglu-Bourbaki theorem and deduce that, if (E, ρ) is a seminormed space, then the closed unit ball (w.r.t. the operator norm ρ') is compact in E'_σ.