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TOPOLOGICAL VECTOR SPACES II–WS 2019/20

Recap Sheet 6

This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** *need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Thursdays 11:45–13:15 in room F408.*

1) Recall the definition of a tensor product of two vector spaces and the construction used to show its existence.

2) Give an example of a tensor product of two vector spaces E and F and an element $w \in E \otimes F$ which can **not** be written as $w = e \otimes f$ for some $e \in E, f \in F$.

3) Recall the universal property and the uniqueness up to isomorphism of a tensor product of two vector spaces.

Let E, F and G be locally convex t.v.s..

4) Recall the definition of π–topology on the tensor product $E \otimes F$ and characterize this topology both in terms of neighbourhoods of the origin and in terms of seminorms.

5) Give a necessary and sufficient condition for $E \otimes_\pi F$ to be Hausdorff. How can the description of the π–topology by seminorms be exploited to prove this result?

6) Characterize the π–topology by a universal property and use this characterization to identify the topological dual $(E \otimes_\pi F)'$ with the space of continuous bilinear functionals on $E \times F$.

7) Recall the definition of an equicontinuous set of linear maps between two t.v.s. and how this differs from a set of continuous linear maps between the same spaces.

8) Recall the definition of Σ–Γ–topology on the space $B(E, F; G)$ of continuous bilinear maps from $E \times F$ to G. Is it a Hausdorff topology?

9) How is the concept of Σ–Γ–topology used to define the ε–topology on $E \otimes F$?

10) Are the π–topology and the ε–topology on $E \otimes F$ comparable?