Universität Konstanz
Fachbereich Mathematik und Statistik
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TOPOLOGICAL VECTOR SPACES–SS 2017
Exercise Sheet 4

This assignment is due by Friday the 26th of May by 11:45 and will be discussed in the tutoral next Tuesday the 30th of May at 13:30 in D404. Please, hand in your solutions in postbox 15 near F411.

1) Prove that a topological space is (T1) if and only if every singleton is closed.

2) Let X be a t.v.s.. Show that the following relations hold:

a) If $A \subset X$, then $\overline{A} = \bigcap_{V \in \mathcal{F}(o)} (A + V)$ where $\mathcal{F}(o)$ is the family of all neighbourhoods of the origin.

b) If $A \subset X$ and $B \subset X$, then $\overline{A} + \overline{B} \subseteq \overline{A + B}$.

c) The intersection of all neighbourhoods of the origin o of X is a vector subspace of X, that is $\overline{\{o\}}$.

3) Let X be the Cartesian product of a family $\{X_i : i \in I\}$ of t.v.s. endowed with the corresponding product topology. Show that:

a) X is a t.v.s.

b) X is Hausdorff if and only if each X_i is Hausdorff.

Does b) hold for any Cartesian product of a family of topological spaces (not necessarily t.v.s.) endowed with the product topology? Justify your answer.

4) Let f, and g be two continuous mappings of a topological space X into a Hausdorff t.v.s. Y. Then:

a) The set A in which f and g coincide, i.e. $A := \{x \in X : f(x) = g(x)\}$, is closed in X.

b) If f and g are equal on a dense subset B of X , then they are equal everywhere in X.