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TOPOLOGICAL VECTOR SPACES II–WS 2017/18

Exercise Sheet 1

This exercise sheet aims to assess your progress and to explicitly work out more details of some of the results proposed in the previous lectures. Please, hand in your solutions in postbox 13 near F411 by Friday the 10th of November at noon. The solutions to this assignment will be discussed in the tutorial on Wednesday the 15th of November (13:30–15:00) in D404.

1) Show the following statements which establish three methods to construct metrizable t.v.s.

a) If X is a metrizable t.v.s., then any linear subspace Y of X equipped with the induced topology is also a metrizable t.v.s..

b) If $\{X_j\}_{j\in J}$ is a countable family of metrizable t.v.s., then $\prod_{j\in J} X_j$ equipped with the product topology is also a metrizable t.v.s..

c) If X is a metrizable t.v.s. and $Y \subset X$ is a closed linear subspace, then the quotient space X/Y equipped with the quotient topology is metrizable t.v.s..

2) Prove the following general properties of metrizable t.v.s. (corresponding respectively to Propositions 1.1.6 and 1.1.7 in the lectures notes).

a) A metrizable t.v.s. X is complete if and only if X is sequentially complete.

b) Let X be a metrizable t.v.s. and Y be any t.v.s. (not necessarily metrizable). A mapping $f : X \to Y$ (not necessarily linear) is continuous if and only if it is sequentially continuous.

3) Consider the following space

$$\ell_1 := \{x = (x_i)_{i\in\mathbb{N}} \subset \mathbb{R} : \sum_{i=1}^{\infty} |x_i| < \infty\}$$

endowed with the topology τ defined by the family of seminorms $\{p_n : n \in \mathbb{N}\}$, where for each $n \in \mathbb{N}$:

$$p_n(x) := \sum_{i=1}^{n} |x_i|, \qquad \forall x = (x_i)_{i\in\mathbb{N}} \in \ell_1.$$

Show that the t.v.s. (ℓ_1, τ) is metrizable but it is not a Baire space.