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TOPOLOGICAL VECTOR SPACES II–WS 2017/2018

Exercise Sheet 2

This exercise sheet aims to assess your progress and to explicitly work out more details of some of the results proposed in the previous lectures. Please, hand in your solutions in postbox 13 near F411 by Friday the 24th of November at noon. The solutions to this assignment will be discussed in the tutorial on Wednesday the 29th of November (13:30–15:00) in D404.

1) Show that an $LF-$space E is a Baire space if and only if it is a Fréchet space.

2) Let E, F be two $LF-$spaces defined by the sequences $\{E_m\}_{m\in\mathbb{N}}$ and $\{F_n\}_{n\in\mathbb{N}}$, respectively. Prove the following statements:

- **a)** If $u : E \to F$ is a continuous linear map, then for any $m \in \mathbb{N}$ there exists $n \in \mathbb{N}$ such that $u(E_m) \subseteq F_n$.
- **b)** If u is a topological isomorphism of E into F, then for any $n \in \mathbb{N}$ there exists $m \in \mathbb{N}$ such that $u^{-1}(F_n) \subseteq E_m$

3) Let $(E, \|\cdot\|)$ be a normed space. For every $k \in \mathbb{N}_0$, let E_k be a linear subspace of E of dimension k, such that $E_k \subseteq E_{k+1}$. Let E_∞ be the union of all the E_k's equipped with the $LF-$topology defined by means of the sequence $\{E_k\}_{k\in\mathbb{N}_0}$. Let $\{\varepsilon_k\}_{k\in\mathbb{N}_0}$ be a decreasing sequence of positive real numbers converging to 0 and set

$$V := \{x \in E_\infty : x \notin E_k \Rightarrow \|x\| < \varepsilon_k,\, k \in \mathbb{N}_0\}.$$

Prove that V is not a neighbourhood of the origin in E_∞.