Universität Konstanz
Fachbereich Mathematik und Statistik
Dr. Maria Infusino
Patrick Michalski

TOPOLOGICAL VECTOR SPACES II–WS 2017/2018

Exercise Sheet 5

This exercise sheet aims to assess your progress and to explicitly work out more details of some of the results proposed in the previous lectures. Please, hand in your solutions in postbox 13 near F411 by Friday the 19th of January at noon. The solutions to this assignment will be discussed in the tutorial on Wednesday the 24th of January (13:30–15:00) in D404.

1) Let E be a locally convex metrizable t.v.s.. Prove that if E is not normable, then E cannot have a countable basis of bounded sets in E (recall Definition 2.2.3 in the lecture notes).

2) Show that a bounded linear map from an LF-space into an arbitrary t.v.s. is always continuous.

3) Prove that the following properties hold for polars of subsets of a t.v.s. E.

a) The polar A° of a subset A of E is a convex balanced subset of E'.

b) If $A \subseteq B \subseteq E$, then $B^\circ \subseteq A^\circ$.

c) $(\rho A)^\circ = (\frac{1}{\rho})A^\circ, \forall \rho > 0, \forall A \subseteq E$.

d) $(A \cup B)^\circ = A^\circ \cap B^\circ, \forall A, B \subseteq E$.

e) If A is a cone in E, then $A^\circ = \{x' \in E' : \forall x \in A : \langle x', x\rangle = 0\}$ and A° is a linear subspace of E'.