Universität Konstanz
Fachbereich Mathematik und Statistik
Dr. Maria Infusino
Patrick Michalski

TOPOLOGICAL VECTOR SPACES II–WS 2017/18

Recap Sheet 2

This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** *need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Thursdays 2-3 pm in room F408.*

1) Do you know a topology that turns $\mathcal{C}^k(\Omega)$ ($\Omega \subseteq \mathbb{R}^d$ open, $k, d \in \mathbb{N}$) into a Fréchet space?

2) Define two topologies on the Schwarz space $\mathcal{S}(\mathbb{R}^d)$ ($d \in \mathbb{N}$) which make it into a t.v.s. and compare them.

3) What is the relation between Fréchet and Baire spaces?

4) Given a family of Fréchet spaces is their product (equipped with the product topology) a Fréchet space?

5) Recall the definition of the inductive topology τ_{ind} on a vector space E. Why does this topology turn E into a l.c.t.v.s.? Is (E, τ_{ind}) a Hausdorff t.v.s.?

6) Recall the definition of LF-space. How can LF-spaces be defined in terms of inductive limits? List at least two examples of LF-spaces.

7) Recall the definitions of finest locally convex topology and finite topology (made in the previous course). Consider the space of polynomials in a single variable $\mathbb{R}[x]$ and compare the finest locally convex, the finite and the inductive limit topology on this space.

8) Let (E, τ_{ind}) be a LF-space with defining sequence $\{(E_n, \tau_n) : n \in \mathbb{N}\}$. Give a criterion for a linear form on E to be continuous.

9) Let (E, τ_{ind}) be a LF-space with defining sequence $\{(E_n, \tau_n) : n \in \mathbb{N}\}$. What can you say about the subspace topology on E_n induced by τ_{ind}?

10) Is every LF-space also a Fréchet space? Justify your answer with a sketch of a proof or a counterexample.