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TOPOLOGICAL VECTOR SPACES II–WS 2017/18

Recap Sheet 3

This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** *need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Thursdays 2-3 pm in room F408.*

1) Recall the definition of projective topology on a vector space. List at least two examples of projective topologies.

2) Give a criterion for a projective topology to be Hausdorff. Use this criterion to reprove the fact, that the product of Hausdorff spaces is itself a Hausdorff space.

3) Compare the concepts of inductive and projective topology.

4) Do you know a vector space which can be equipped with both an inductive and a projective topology? What are the defining sequences?

5) Can you give an example of a $\mathcal{C}_c^\infty$ function on $\mathbb{R}^d$ ($d \in \mathbb{N}$)?

6) Recall the construction of the function f_ε introduced in Lemma 1.5.3 for some $f \in \mathcal{C}_c(\mathbb{R}^d)$, where $\varepsilon > 0$ and $d \in \mathbb{N}$. Why is f_ε a useful function?

7) Recall the definition of dense and sequentially dense. When do both notions coincide?

8) Which sequentially dense subspaces of $\mathcal{C}^k(\Omega)$ ($\Omega \subseteq \mathbb{R}^d$ open, $k, d \in \mathbb{N}$) do you know? Which of them are actually dense?

9) Recall the definition of accumulation point of a filter of a topological space. What is the relation between accumulation points and limit points of a filter of a t.v.s.?

10) Give a criterion for a Hausdorff space to be compact in terms of accumulation points.