



TOPOLOGICAL VECTOR SPACES II–WS 2017/18

Recap Sheet 5

*This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Thursdays 2-3 pm in room F408.*

- 1) Prove that the polar of a bounded subset of a t.v.s. is absorbing. Why is this a crucial observation?
- 2) Recall the definition of Σ -topology on the topological dual of a t.v.s. and list at least three important examples.
- 3) Is a Σ -topology on the topological dual of a t.v.s. a locally convex topology? Justify your answer with a proof or give a counterexample.
- 4) Give a sufficient condition for a Σ -topology on the topological dual of a t.v.s. to be Hausdorff.
- 5) Let E be a t.v.s. and for $x \in E$ consider the map $v_x : E' \rightarrow \mathbb{K}$, $x' \mapsto \langle x', x \rangle$. Give an example of a class of topologies on E' for which v_x is continuous for all $x \in E$.
- 6) When can a t.v.s. E be regarded as the topological dual of its weak dual E'_σ ?
- 7) Which condition ensures that the topological dual of a t.v.s. contains a non-zero element? Give an example of a t.v.s. whose topological dual is trivial.
- 8) State the Banach-Alaoglu-Bourbaki theorem and deduce that, if (E, ρ) is a normed space, then the closed unit ball (w.r.t. the operator norm ρ') is compact in E'_σ .
- 9) Recall the definition of tensor product of vector spaces and state the universal property of such tensor products.
- 10) Give an example of a tensor product of two vector spaces E and F and an element $w \in E \otimes F$ which can **not** be written as $w = e \otimes f$ for some $e \in E, f \in F$.