



TOPOLOGICAL VECTOR SPACES II–WS 2017/18

Recap Sheet 6

*This recap sheet aims to self-assess your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please do not hesitate to attend Maria's office hours on Thursdays 2-3 pm in room F408.*

Let E and F, G be locally convex t.v.s..

- 1) Which construction is used to show the existence of a tensor product of two vector spaces?
- 2) Recall the definition of π -topology on the tensor product $E \otimes F$ and characterize this topology in terms of neighbourhoods of the origin.
- 3) Give a necessary and sufficient condition for $E \otimes_{\pi} F$ to be Hausdorff. How can the description of the π -topology by seminorms be exploited to prove this result?
- 4) Characterize the π -topology by a universal property and use this characterization to identify the topological dual $(E \otimes_{\pi} F)'$ with the space of continuous bilinear functionals on $E \times F$.
- 5) Recall the definition of an equicontinuous set of functions. How this generalization of continuity differs from the notion of uniform continuity?
- 6) Show that any equicontinuous subset in the topological dual E' is bounded in E'_{σ} .
- 7) How can the set of all equicontinuous functions of E' be used to define a topology on the topological dual of E' ?
- 8) Recall the definition of Σ - Γ -topology on $B(E, F; G)$. Is it a Hausdorff topology?
- 9) How is the concept of Σ - Γ -topology used to define the ε -topology on $E \otimes F$?
- 10) Are the π - and the ε -topology on $E \otimes F$ comparable?