Universität Konstanz
Fachbereich Mathematik und Statistik
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TOPOLOGICAL VECTOR SPACES–WS 2018/19

Exercise Sheet 2

This assignment is due by November 27th by 15:00 and will be discussed in the following tutorials: November 30th (Group A) and December 3rd (Group B). Please, hand in your solutions in the Postbox 11 (near F411). If you should have any problem in solving the exercises, please take advantage of the Fragestunde *on Wednesday 13:30–14:30 in room F408.*

1) Prove that a topological space is (T1) if and only if every singleton is closed.

2) Let X be a t.v.s.. Show that the following relations hold:

a) If $A \subset X$, then $\overline{A} = \bigcap_{V \in \mathcal{F}(o)} (A + V)$, where $\mathcal{F}(o)$ is the family of all neighbourhoods of the origin.

b) If $A \subset X$ and $B \subset X$, then $\overline{A} + \overline{B} \subseteq \overline{A + B}$.

c) The intersection of all neighbourhoods of the origin o of X is a vector subspace of X, that is $\overline{\{o\}}$.

3) Let X be the Cartesian product of a family $\{X_i : i \in I\}$ of t.v.s. endowed with the corresponding product topology. Show that:

a) X is a t.v.s..

b) X is Hausdorff if and only if each X_i is Hausdorff.

Does b) hold for any Cartesian product of a family of topological spaces (not necessarily t.v.s.) endowed with the product topology? Justify your answer.

4) Let f and g be two continuous mappings of a topological space X into a Hausdorff t.v.s. Y. Then:

a) The set A in which f and g coincide, i.e. $A := \{x \in X : f(x) = g(x)\}$, is closed in X.

b) If f and g are equal on a dense subset B of X , then they are equal everywhere in X.

5) Prove the following statements.

a) If M is a linear dense subspace of a t.v.s X, then the quotient topology on X/M is the trivial topology.

b) If X is a Hausdorff t.v.s., then any linear subspace M of X endowed with the corresponding subspace topology is itself a Hausdorff t.v.s..

c) Give an example of a t.v.s X which is not Hausdorff and of a linear subspace $M \neq \{o\}$ of X such that M endowed with the subspace topology is instead a Hausdorff t.v.s..