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TOPOLOGICAL VECTOR SPACES–WS 2018/19

Recap Sheet 2

This recap sheet aims at self-assessing your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** *need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please take advantage of the* Fragestunde *on Wednesday 13:30–14:30 in room F408.*

1) Give the definition of continuous, open and closed map between two topological spaces. Provide a different example for each of them.

2) Clarify the following terms: basis of a topology, basis of a filter, basis of neighbourhoods of a point, basis of a vector space.

3) Let $f : X \to Y$ be a continuous map between the topological spaces (X, τ_X) and (Y, τ_Y). Let $\mathcal{B}$ be a basis of τ_X and consider the following collection $f(\mathcal{B}) := \{f(B) : B \in \mathcal{B}\}$ of subsets of Y. Show that if f is surjective and open, then $f(\mathcal{B})$ is a basis of τ_Y.

4) Recall the definition of homeomorphism between topological spaces. Provide an example and a counterexample.

5) Characterize the notion of comparable topologies on the same set in terms of continuity.

6) Given a set X of finite cardinality, list all the topologies you know that make X into a Hausdorff space. Provide also an example of an uncountable topological space which is not Hausdorff.

7) Give the definition of real topological vector space (t.v.s.) and characterize its filter of neighbourhoods of the origin. How can we retrieve the topology of t.v.s. knowing only its filter of neighbourhoods of the origin?

8) Show that the topology of a t.v.s. is always translation invariant. Does the converse statement hold? Justify your answer with a proof or with a counterexample.

9) Recall the definition of absorbing and balanced subset of a vector space. Give an example of a subset which has exactly one of these properties, both or neither of them.

10) Characterize the notion of Hausdorff t.v.s. and use this result to provide an example of topological space which is not a t.v.s..