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TOPOLOGICAL VECTOR SPACES–WS 2018/19

Recap Sheet 3

This recap sheet aims at self-assessing your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** *need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please take advantage of the* Fragestunde *on Wednesday 13:30–14:30 in room F408.*

1) Provide an example of a t.v.s. having a basis of neighbourhoods of the origin consisting of non-balanced sets. For the same t.v.s., provide a basis of balanced neighbourhoods of the origin.

2) Given a linear subspace M of a vector space X, define the corresponding quotient space. List the properties of the quotient map $\phi : X \to X/M$.

3) Given an equivalence relation $\sim$ on a topological space X, define the corresponding quotient topology on $X/\sim$. List the properties of the quotient map $\phi : X \to X/\sim$.

4) Given a linear subspace M of a t.v.s. X, endow the quotient space X/M with the corresponding quotient topology and list the properties of the quotient map $\phi : X \to X/M$.

5) Provide an example of a quotient of a t.v.s. for which the corresponding quotient map is not open and for which is not closed.

6) Characterize the closed linear subspaces of a t.v.s. in terms of quotients. Give an example of a quotient of a non-Hausdorff t.v.s. which is Hausdorff.

7) Give a sufficient condition for a continuous linear map between two t.v.s. to have a closed kernel. Provide an example showing that such a condition is not necessary.

8) Highlight the difference between the algebraic and the topological dual of a t.v.s.. Do you know any t.v.s. whose algebraic and topological dual coincide?

9) Define the notions of Cauchy sequence and Cauchy filter in a t.v.s.. What is the relation between them?

10) Is every converging sequence in a t.v.s. a Cauchy sequence? Justify your answer!