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TOPOLOGICAL VECTOR SPACES–WS 2018/19

Recap Sheet 4

This recap sheet aims at self-assessing your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** *need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please take advantage of the* Fragestunde *on Wednesday 13:30–14:30 in room F408.*

1) Give an example of a t.v.s. which is sequentially complete but not complete. Do you know a sufficient condition for these two notions to coincide in a t.v.s.?

2) Recall from your previous courses in Analysis how to construct the completion of a metric space.

3) Do you know any class of t.v.s. in which closedness and completeness are equivalent properties for subsets?

4) Characterize the continuity of a non-identically zero linear functional L on a t.v.s. in terms of its null space $\ker(L)$.

5) Recall the definition of compact topological space and show that the intersection of two compact sets in a topological space is compact.

6) Give the definition of locally compact topological space and compare it with the notion of compactness of a t.v.s.. Do you know a sufficient condition for a topological space (not necessarily a t.v.s.) to be locally compact?

7) List as many as possible properties of finite dimensional Hausdorff t.v.s.. In particular, point out the relation between the dimension of a t.v.s. and its local compactness.

8) State the Tychonoff theorem for finite dimensional t.v.s. and give an example of two distinct topologies on the same infinite dimensional vector space that make it into a Hausdorff t.v.s..

9) Give the definition of convex, absolutely convex and barrelled subset of a t.v.s., providing a different example for each of these notions.

10) Do you know a class of t.v.s. in which every neighbourhood of the origin is contained in (resp. contains) a barrelled neighbourhood of the origin?