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TOPOLOGICAL VECTOR SPACES–WS 2018/19

Recap Sheet 5

This recap sheet aims at self-assessing your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** *need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please take advantage of the* Fragestunde *on Wednesday 13:30–14:30 in room F408.*

1) Recall the definition of locally convex t.v.s. and provide all the characterizations you know in terms of basis of neighbourhoods of the origin.

2) Provide two topologies on the same vector space such that one makes it into a locally convex t.v.s. and the other into a t.v.s. which is not locally convex.

3) Give the definition of a seminorm on a vector space and give three examples of seminorms on the space of continuous functions on the real line.

4) List the main properties of the semiball associated to a seminorm on a vector space (resp. on a t.v.s.).

5) Establish the connection between the notions of Minkowski functional and absorbing absolutely convex subsets of a vector space.

6) Characterize locally convex t.v.s. in terms of seminorms.

7) Give a criterion to compare two locally convex topologies on the same t.v.s.. Use this result to show that the topology of a locally convex t.v.s. does not change if we close the generating family of seminorms under taking the maximum of finitely many of its elements.

8) What is a directed family of seminorms? State one advantage of choosing a directed family of seminorms among all the families of seminorms inducing the same locally convex topology on a t.v.s..

9) Define the concept of separating family of seminorms on a vector space, providing at least one example of such a family.

10) Characterize Hausdorff locally convex t.v.s. in terms of seminorms and give two examples of spaces of functions having these properties.