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TOPOLOGICAL VECTOR SPACES–WS 2018/19

Recap Sheet 6

This recap sheet aims at self-assessing your progress and to recap some of the definitions and concepts introduced in the previous lectures. You do **not** *need to hand in solutions, but please try to answer as many questions as you can since this is a very good training in preparation of your final exam. If you should have any problem, please take advantage of the* Fragestunde *on Wednesday 13:30–14:30 in room F408.*

1) Give an example of a topology on the Schwarz space on $\mathbb{R}^d$ which makes it into a locally convex Hausdorff t.v.s..

2) Do you know a class of t.v.s. whose algebraic dual coincides with the topological one?

3) Recall the definition of finest locally convex topology on a vector space and characterize it both via neighbourhoods of the origin and seminorms.

4) Which is the finest locally convex topology on a countable dimensional real vector space? Give an example of such a vector space and explicitly write down the form of a generic open set in the finest locally convex topology on this space.

5) Characterize the continuity of a non-zero linear functional on a locally convex t.v.s. in terms of a given family of seminorms generating its topology. Distinguish the case when the given generating family of seminorms is directed from the case when it is not.

6) State both the analytic and the geometric form of the Hahn-Banach theorem given in the lecture.

7) Provide an example of t.v.s. X whose topological dual X' is non-trivial, i.e. $X' \neq \{o\}$.

8) When is it possible to separate two disjoint non-empty convex subsets A and B of a t.v.s. X by a hyperplane? Highlight the cases when the hyperplane can be chosen to strictly separate A and B.

9) Do you know a class of t.v.s. in which a closed convex cone can be always separated from a point not belonging to that cone? Give an example of convex cone in $\mathbb{R}[x]$ having this property, specifying the considered topology on $\mathbb{R}[x]$.

10) Describe the closure of a non-empty convex cone in a vector space X endowed with the finest locally convex topology φ, using the elements of the topological dual X' of (X, φ).