Fachbereich Mathematik und Statistik
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Universität Konstanz

Real Algebraic Geometry I

Exercise Sheet 5
Counting roots

Exercise 17
(4 points)

(a) How many distinct orderings on $\mathbb{Q}(\sqrt{2}+\sqrt{3})$ extend the ordering on $\mathbb{Q}$?

(b) How many distinct orderings on $\mathbb{Q}(\pi)$ extend the ordering on $\mathbb{Q}$?

Justify your answers!

Exercise 18
(4 points)
Let $(K,<)$ be an ordered field such that for any $f(\mathrm{x}) \in K[\mathrm{x}]$ the intermediate value property holds, i.e. for any $a, b \in K$ with $a < b$

$$f(a) < 0 < f(b) \Longrightarrow \exists c \in \,]a, b[: f(c) = 0.$$

Show that K is real closed.

Exercise 19
(4 points)
Let R be a real closed field.

(a) Show that the polynomial $f(\mathrm{x}) = \mathrm{x}^5 - 4\mathrm{x}^2 + 4\mathrm{x} - 1$ has no root in R which is greater than 1.

(b) Let $g(\mathrm{x}) = \mathrm{x}^3 - b \in R[\mathrm{x}]$ with $b \neq 0$. Compute the number of roots of g in R.

(c) Construct a polynomial $h(\mathrm{x}) \in R[\mathrm{x}]$ consisting of 2 monomials which has exactly 3 distinct roots in R.

Exercise 20
(4 points)
Let R be a real closed field and let $f(\mathrm{x}) = \mathrm{x}^3 + 6\mathrm{x}^2 - 16 \in R[\mathrm{x}]$.

(a) Compute the Sturm sequence of f.

(b) Show that f has three distinct roots in $[-6, 2]$.

(c) Denote the roots of f by $\alpha_1 < \alpha_2 < \alpha_3$. Show that $\alpha_1 \in [-6, -5]$, $\alpha_2 \in [-3, -1]$ and $\alpha_3 \in [1, 2]$.

Please hand in your solutions by ***Thursday, 29 November 2018, 08:15h*** *(postbox 16 in F4).*